International Journal of Theoretical Physics, Vol. 23, No. 6, 1984

Trilocal Structures. V. Wave Equations

Roger E. Clapp, Evelyn W. Mack, Fay Simons, and Joseph A. Wolf, Jr.

Basic Research Associates, Incorporated, Cambridge, Massachusetts 02138

Received January 19, 1982

In addition to the usual centroid-time wave equation, a trilocal structure will need to satisfy two relative-time wave equations. When the trilocal wave function is expanded in tree functions, each of the three wave equations becomes an infinite matrix equation, but when the four auxiliary conditions (defined in earlier articles in this series) are introduced, each wave equation reduces to a set of 16 linear homogeneous equations in 16 unknown expansion coefficients (the first 16 coefficients in the tree expansion). The 48 linear equations, in the 16 unknown C_j, are given explicitly. Every 16-by-16 determinant, formed from any 16 of these 48 linear homogeneous equations, must vanish if the trilocal structure is to be an acceptable solution; this requirement will be used in later calculations.

1. INTRODUCTION

The earlier articles in this set (Clapp et al., 1980, 1979, 1981a, and 1981b), which will be referred to here as I, II, III, and IV, introduced notation, equations, and expansion functions for the trilocal system. The auxiliary conditions that were introduced permitted the higher coefficients in the infinite expansion to be replaced by linear combinations of the 16 leading coefficients.

This explicit reduction technique was applied to the expansion in terms of "tree" functions, though a similar reduction procedure would be applicable to the analogous expansion in terms of the "bowl" functions in II (together with the corresponding momentum-dependent bowl functions).

The initial tree function, as given in (III.2.1), is

$$\varphi_1 = N_0 j_{0,0}\left[(+)^{\tau\, 2b}(1)+(-)^{\tau\, 2c}(1)\right] \qquad (1)$$

0020-7748/84/0600-0499$03.50/0

where the normalization factor N_0 is given by

$$N_0 = \kappa^{3/2}/4 \tag{2}$$

Here the wave number κ is the only adjustable parameter in the theory. It specifies the height of the Fermi sea filling the vacuum, as discussed in detail by Clapp (1980). As we will see, however, κ also serves as the mass unit (in wave number units) for the elementary particle mass spectrum. In (2) $\kappa^{3/2}$ gives the tree function (1) the dimension of $(\text{length})^{-3/2}$, so that its square can have the dimension of a volume density or probability density, in accordance with the usual interpretation of a wave function in quantum mechanics.

The expression in brackets in (1) gives the combined τ-spin and σ-spin dependence of this initial tree function. This expression is antisymmetric with respect to P_{23}, the interchange of the two quanta labeled 2 and 3. The antisymmetry follows from the relationships

$$\begin{aligned} P_{23}(+)^\tau &= (+)^\tau, \qquad & P_{23}{}^{2b}(1) &= -{}^{2b}(1) \\ P_{23}(-)^\tau &= -(-)^\tau, \qquad & P_{23}{}^{2c}(1) &= {}^{2c}(1) \end{aligned} \tag{3}$$

which can be verified from the definitions of $(+)^\tau$ and $(-)^\tau$ in (III.2.2) and the definitions of ${}^{2b}(1)$ and ${}^{2c}(1)$ in (II.2.9) and (II.2.10).

The "radial" factor $j_{0,0}$ is the product of two spherical Bessel functions,

$$j_{0,0} = j_0(\kappa_r r)\, j_0(\kappa_\rho \rho) \tag{4}$$

and is unchanged by the operator P_{23}, as can be seen from Figure 1 of III. This operator leaves the vector $\mathbf{r}$ unchanged while reversing the vector $\boldsymbol{\rho}$. The magnitudes r and ρ are accordingly unaltered by P_{23}, and since only the magnitudes enter into (4) it is clear that $j_{0,0}$ is unchanged.

The tree function (1), like all the other tree functions in the trilocal expansion, singles out one of the three quanta (quantum 1) for special treatment. However, the tree function φ_1 can be converted to a fully antisymmetric function $\bar{\varphi}_1$ through averaging over cyclic permutations:

$$\bar{\varphi}_1 = (1/3)(1 + P_{123} + P_{213})\varphi_1 \tag{5}$$

where P_{123} and P_{213} are the two possible cyclic permutations of three objects. It is easily verified that $\bar{\varphi}_1$ is antisymmetric to any one of the three pair exchanges, P_{12}, P_{23}, and P_{13}.

For most of the analytic work, we will be using the not-fully-antisymmetric functions such as φ_1, with the understanding that eventually these will be replaced by the fully antisymmetric functions such as $\bar{\varphi}_1$. A few of the expansion functions will need special treatment. These are functions φ_j which are nonvanishing at the central point,

$$\mathbf{r} = \boldsymbol{\rho} = 0 \tag{6}$$

but which have a spin dependence which leads to the vanishing of the fully antisymmetric function $\bar{\varphi}_j$ constructed as in (5), at the central point (6). There are only a few such functions, and they do not belong as part of a well-behaved trilocal structure. Accordingly, for these few functions the corresponding coefficients C_j should vanish, and requiring that they vanish is part of the specification of a valid trilocal structure. This requirement can be thought of as an inner boundary condition, complementing the outer boundary condition which has been given in (I.2.11).

A valid trilocal structure should also satisfy the centroid-time wave equation (I.2.8a), and two relative-time wave equations. The latter are equivalent to (I.2.8b) and (I.2.8c), but will be given later in a somewhat different form.

Each of the three wave equations can be expressed as an infinite matrix equation relating the coefficients C_j in the trilocal expansion. By the use of the reduction equations in III and IV, each of the higher coefficients can be written as a linear combination of the first 16, $C_1, C_2, \ldots, C_{16}$. Accordingly, each of the three infinite matrix equations reduces to 16 linear homogeneous relationships among these first 16 coefficients.

The present paper will be directed to the construction of these 48 linear equations.

2. CENTROID-TIME WAVE EQUATION

The centroid-time wave equation (I.2.8a) can be written in the form

$$0 = \left(H_k + H_{r,\rho} - w\right)\Phi \tag{7}$$

where H_k and $H_{r,\rho}$ are given by

$$H_k = (1/9)(\boldsymbol{\sigma}^s\cdot\mathbf{k} - 3P^{\tau}\boldsymbol{\sigma}^b\cdot\mathbf{k} + \boldsymbol{\sigma}^c\cdot\mathbf{k}) \tag{8}$$

$$H_{r,\rho} = (1/6i\kappa)\big(2\boldsymbol{\sigma}^s\cdot\nabla_r - 4P^{\tau}\boldsymbol{\sigma}^s\cdot\nabla_\rho + 3P^{\tau}\boldsymbol{\sigma}^b\cdot\nabla_r + 2\boldsymbol{\sigma}^c\cdot\nabla_r + 2P^{\tau}\boldsymbol{\sigma}^c\cdot\nabla_\rho\big) \tag{9}$$

The σ-spin operators are

$$\sigma^s = \sigma_1 + \sigma_2 + \sigma_3, \qquad \sigma^b = \sigma_2 - \sigma_3, \qquad \sigma^c = 2\sigma_1 - \sigma_2 - \sigma_3 \tag{10}$$

while the τ-spin operator P^τ is defined through

$$P^\tau(+)^\tau = (-)^\tau, \qquad P^\tau(-)^\tau = (+)^\tau \tag{11}$$

The wave function Φ is the expansion

$$\Phi = C_1\varphi_1 + C_2\varphi_2 + \cdots \tag{12}$$

where $\varphi_1, \varphi_2, \ldots,$ are the tree functions introduced in III and IV.

Operation by $H_{r,\rho}$ upon the rest-system tree functions gives the results assembled in Appendices B and D of III. Operation by H_k upon the first 16 rest-system functions gives the results which appear here in Appendix A. The momentum-dependent functions which are generated by this operator are given in the notation used in IV.

Operation once again by H_k upon the functions on the right-hand sides of the equations in Appendix A will give further momentum-dependent functions, together with rest-system functions including the rest-system functions on the left-hand sides of the equations in Appendix A. The matrix elements that are generated by these further operations are found to be symmetric with respect to the main diagonal.

That is, the operator H_k, when applied to members of the expansion system φ_j, including rest-system functions and momentum-dependent functions, can be represented by an infinite matrix which is symmetric about its main diagonal. This means that the operator H_k is Hermitian. The operator $H_{r,\rho}$ is similarly Hermitian.

The wave equation (7), when the expansion (12) is inserted and the explicit algebraic operations are carried out, becomes a large relationship connecting (linearly) the expansion functions φ_j. The terms in this equation can be rearranged to group all terms involving φ_1, and all terms involving φ_2, and so forth. These functions are all linearly independent, and are actually all orthogonal, though this will not be proved here. As a consequence, each such grouping must vanish separately. That is, the terms in the expression which multiplies φ_1 must add to zero, the terms in the expression which multiplies φ_2 must add to zero, and so on for each of the expansion functions φ_j.

From the terms containing φ_1, within the expanded form of the wave equation (7), we obtain the following relationship connecting the expansion coefficients C_j:

$$0 = -wC_1 + (\kappa_r/\kappa)[-(6^{1/2}/2)C_3 + (1/2)C_5 + (3^{1/2}/2)C_6]$$
$$+ (\kappa_\rho/\kappa)[-(6^{1/2}/3)C_4 - C_7 + (3^{1/2}/3)C_8]$$
$$+ k[-(1/3)C_5^{0,0,k}] \qquad (13)$$

Furthermore, we can use (IV.133) to express $C_5^{0,0,k}$ as a linear combination of C_5, C_7, and C_{13}, with coefficients involving the parameters ν, ν', ν'', and γ, defined through (IV.8–11). After this substitution for $C_5^{0,0,k}$ we find that (13) takes the form of a linear homogeneous equation in certain of the leading coefficients $C_1,\ldots,C_{16}$. The explicit form is given as the first equation in Appendix B.

Similarly, from the terms containing φ_2 in the expanded form of (7) we can construct an equation which reduces to the second equation in Appendix B. In this way, continuing, we can construct all 16 of the equations in Appendix B. These equations make use of the reduction equations given earlier in III and IV, by which any one of the expansion coefficients in the general tree expansion can be expressed as a linear combination of the first 16 C_j.

The 16 linear homogeneous equations in 16 unknown coefficients C_j can only be satisfied if the 16-by-16 secular determinant vanishes. The vanishing of this determinant is one of the conditions which must be satisfied by a valid trilocal solution.

3. RELATIVE-TIME WAVE EQUATIONS

The trilocal wave function depends upon three time variables. As shown in (I.2.1), these can be taken as the three individual times t_1, t_2, t_3, associated with the three constituent quanta. However, for the description of a composite trilocal structure it is more appropriate to use the centroid time, T, given by

$$T = (t_1 + t_2 + t_3)/3 \qquad (14)$$

and the two relative times, t_r and t_ρ, given by

$$t_r = (2t_1 - t_2 - t_3)/2, \qquad t_\rho = (t_2 - t_3) \qquad (15)$$

These were defined earlier in (I.2.2).

Partial differentiation with respect to the centroid time and the two relative times is shown in (I.2.3). The Schrödinger relationship between

$\partial/\partial T$ and energy is expressed, as in (I.2.10), through

$$\frac{1}{ic}\frac{\partial}{\partial T} \to -\kappa w \tag{16}$$

This defines the dimensionless energy parameter w which appears in (7). In analogy with (16), we can introduce the dimensionless parameters w_r and w_ρ through

$$\frac{3}{ic}\frac{\partial}{\partial t_r} \to -\kappa w_r, \qquad \frac{2}{ic}\frac{\partial}{\partial t_\rho} \to -\kappa w_\rho P^\tau \tag{17}$$

The specifications (17) represent new conditions which are being placed upon the trilocal wave function. This wave function is now called upon to be an eigenfunction of these two time-differential operators. As will be shown below, these operators commute with the Hamiltonian for the trilocal system, and thus can be specified independently as conserved operators. However, as will be seen, the associated eigenvalues are not entirely independent of other parameters.

As shown in (I.4.1), we can convert (7) into

$$(i\kappa w) = \tau_{1\zeta}\sigma_1\cdot\nabla_1 + \tau_{2\zeta}\sigma_2\cdot\nabla_2 + \tau_{3\zeta}\sigma_3\cdot\nabla_3 \tag{18}$$

Similarly, we can use (17) to construct

$$(i\kappa w_r) = 2\tau_{1\zeta}\sigma_1.\nabla_1 - \tau_{2\zeta}\sigma_2\cdot\nabla_2 - \tau_{3\zeta}\sigma_3\cdot\nabla_3 \tag{19}$$

$$P^\tau(i\kappa w_\rho) = \tau_{2\zeta}\sigma_2\cdot\nabla_2 - \tau_{3\zeta}\sigma_3\cdot\nabla_3 \tag{20}$$

It can be seen by inspection that the operators on the right-hand sides of (19) and (20) commute with each other and with the right-hand side of (18). It can also be established, with the help of (I.2.11), that

$$w^2 - k^2 = 9 - (1/2)w_r^2 - (3/2)w_\rho^2 \tag{21}$$

From (I.4.4), it is evident that the right-hand side of (21) must equal m^2, the square of the (dimensionless) mass of the trilocal structure. That is, we must have

$$m^2 = 9 - (1/2)w_r^2 - (3/2)w_\rho^2 \tag{22}$$

which is accordingly a relationship connecting certain of the parameters characterizing a trilocal structure.

The operator equations (19) and (20) are really eigenvalue equations restricting the trilocal wave function Φ. In place of (19) and (20) we can write

$$0 = (H_{tr} - w_r)\Phi \tag{23}$$

$$0 = (H_{t\rho} - w_\rho)\Phi \tag{24}$$

where H_{tr} and $H_{t\rho}$ are given by

$$\begin{aligned} H_{tr} = (1/6i\kappa)(\quad &\cdot\nabla_r + 4P^\tau\boldsymbol{\sigma}^s\cdot\nabla_\rho - 3P^\tau\boldsymbol{\sigma}^b\cdot\nabla_r S \\ &+ 4\boldsymbol{\sigma}^c\cdot\nabla_r - 2P^\tau\boldsymbol{\sigma}^c\cdot\nabla_\rho) \\ &+ (1/9)(2\boldsymbol{\sigma}^s\cdot\mathbf{k} + 3P^\tau\boldsymbol{\sigma}^b\cdot\mathbf{k} + 2\boldsymbol{\sigma}^c\cdot\mathbf{k}) \end{aligned} \tag{25}$$

$$\begin{aligned} H_{t\rho} = (1/6i\kappa)&(2\boldsymbol{\sigma}^s\cdot\nabla_r - 6\boldsymbol{\sigma}^b\cdot\nabla_\rho - \boldsymbol{\sigma}^c\cdot\nabla_r) \\ &+ (1/9)(-2\boldsymbol{\sigma}^s\cdot\mathbf{k} + \boldsymbol{\sigma}^c\cdot\mathbf{k}) \end{aligned} \tag{26}$$

The action of these operators H_{tr} and $H_{t\rho}$ upon the first 16 expansion functions, $\varphi_1, \varphi_2, \ldots, \varphi_{16}$, is shown in Appendices C and D. The functions introduced by these operations include rest-system functions and functions which are linear in the momentum vector **k**.

Substitution of the expansion (12) into (23), and rearrangement of terms to collect those terms involving each of the expansion functions φ_j, leads to relations among the coefficients C_j, one relation for each function. In the first 16 of these relations, we can make substitutions which replace the higher C_j by linear combinations using the first 16 C_j. What results is the first 16 equations in Appendix E.

A similar substitution of the expansion (12) into (24) leads to relations among the coefficients C_j, which can be reduced to relations among only the first 16 C_j. These latter are included as the second 16 equations in Appendix E.

Each of the 48 equations in Appendices B and E is a linear homogeneous equation among 16 unknown coefficients C_j. For a valid solution to exist, every 16-by-16 determinant that can be formed from any 16 rows of this 48-by-16 matrix must vanish. This requirement severely limits the possible solutions, as will be seen later.

Part of the limitation arises from the restriction of the eigenvalues w_r and w_ρ to explicit forms, which will be given later. These forms satisfy (22), but are more restrictive.

4. DISCUSSION

In a trilocal structure, each of the constituent quanta satisfies a simple, relativistically invariant wave equation, as given in (I.2.1). There are three time variables that appear, one for each quantum.

When we want to look at the structure as a whole, we will use a centroid time which we associate with the structure. This is the average of the three individual constituent times. Differentiation with respect to this centroid time gives the usual wave equation, analogous to the Dirac equation.

But there are then two relative-time variables which need to be included. Differentiation with respect to these relative-time variables leads to two relative-time wave equations which the trilocal structure needs to satisfy.

A distributed trilocal structure cannot hope to be relativistically invariant unless it includes dependence upon three time variables, the centroid time plus two relative-time variables. Each of these time dependencies has its own wave equation.

There are accordingly three wave equations. Each of the three is here translated into 16 linear homogeneous equations in 16 unknown coefficients C_j. The compatibility of these 48 equations requires the vanishing of a large number of 16-by-16 determinants, and this is an important part of the specification of a valid trilocal solution.

Other parts of the specification of a valid trilocal solution will be examined in succeeding papers.

APPENDIX C: 16 ROWS OF THE H_{tr} MATRIX

$$H_k\varphi_1 = k\left[-(1/3)\varphi_5^{0,0,k}\right]$$

$$H_k\varphi_2 = k\left[4(2^{1/2}/9)\varphi_3^{0,0,k} + (5/9)\varphi_6^{0,0,k}\right]$$

$$H_k\varphi_3 = k\left[4(6^{1/2}/27)\varphi_2^{1,0,k} + 5(6^{1/2}/54)\varphi_9^{1,0,k}\right.$$
$$\left. + (30^{1/2}/54)\varphi_{10}^{1,0,k} - 4(3^{1/2}/27)\varphi_{14}^{1,0,k}\right]$$

$$H_k\varphi_4 = k\left[4(6^{1/2}/27)\varphi_2^{0,1,k} + 5(6^{1/2}/54)\varphi_9^{0,1,k}\right.$$
$$\left. + (30^{1/2}/54)\varphi_{10}^{0,1,k} - 4(3^{1/2}/27)\varphi_{14}^{0,1,k}\right]$$

$$H_k\varphi_5 = k\left[-(3^{1/2}/9)\varphi_1^{1,0,k} - (6^{1/2}/9)\varphi_{13}^{1,0,k}\right]$$

$$H_k\varphi_6 = k\left[5(3^{1/2}/27)\varphi_2^{1,0,k} - 4(3^{1/2}/27)\varphi_9^{1,0,k}\right.$$

$$\left. + 4(15^{1/2}/27)\varphi_{10}^{1,0,k} + 5(6^{1/2}/27)\varphi_{14}^{1,0,k}\right]$$

$$H_k\varphi_7 = k\left[-(3^{1/2}/9)\varphi_1^{0,1,k} - (6^{1/2}/9)\varphi_{13}^{0,1,k}\right]$$

$$H_k\varphi_8 = k\left[5(3^{1/2}/27)\varphi_2^{0,1,k} - 4(3^{1/2}/27)\varphi_9^{0,1,k}\right.$$

$$\left. + 4(15^{1/2}/27)\varphi_{10}^{0,1,k} + 5(6^{1/2}/27)\varphi_{14}^{0,1,k}\right]$$

$$H_k\varphi_9 = k\left[4(6^{1/2}/27)\varphi_2^{1,1,k} - 5(6^{1/2}/54)\varphi_{3b}^{1,1,k}\right.$$

$$\left. + 4(3^{1/2}/27)\varphi_{6b}^{1,1,k} + (30^{1/2}/54)\varphi_{15a}^{1,1,k}\right]$$

$$H_k\varphi_{10} = k\left[(2^{1/2}/18)\varphi_3^{2,0,k} + (4/9)\varphi_6^{2,0,k} + (2^{1/2}/6)\varphi_{15}^{2,0,k}\right]$$

$$H_k\varphi_{11} = k\left[(2^{1/2}/18)\varphi_3^{0,2,k} + (4/9)\varphi_6^{0,2,k} + (2^{1/2}/6)\varphi_{15}^{0,2,k}\right]$$

$$H_k\varphi_{12} = k\left[(2^{1/2}/27)\varphi_{3s}^{1,1,k} + (10^{1/2}/54)\varphi_{3c}^{1,1,k}\right.$$

$$\left. + (8/27)\varphi_{6s}^{1,1,k} + 4(5^{1/2}/27)\varphi_{6c}^{1,1,k} + (2^{1/2}/6)\varphi_{15s}^{1,1,k}\right]$$

$$H_k\varphi_{13} = k\left[-(3^{1/2}/9)\varphi_1^{1,1,k} + (6^{1/2}/9)\varphi_{5b}^{1,1,k}\right]$$

$$H_k\varphi_{14} = k\left[5(3^{1/2}/27)\varphi_2^{1,1,k} + 4(3^{1/2}/27)\varphi_{3b}^{1,1,k}\right.$$

$$\left. - 5(6^{1/2}/27)\varphi_{6b}^{1,1,k} + 4(15^{1/2}/27)\varphi_{15a}^{1,1,k}\right]$$

$$H_k\varphi_{15} = k\left[(2^{1/2}/18)\varphi_{9a}^{2,1,k} - (2^{1/2}/6)\varphi_{10b}^{2,1,k}\right.$$

$$\left. + (4/9)\varphi_{14a}^{2,1,k}\right]$$

$$H_k\varphi_{16} = k\left[(2^{1/2}/18)\varphi_{9a}^{1,2,k} + (2^{1/2}/6)\varphi_{10b}^{1,2,k}\right.$$

$$\left. + (4/9)\varphi_{14a}^{1,2,k}\right]$$

APPENDIX B: THE 16-ROW REDUCED CENTROID-TIME WAVE EQUATION

$$0 = -wC_1 + (\kappa_r/\kappa)[-(6^{1/2}/2)C_3 + (1/2)C_5 + (3^{1/2}/2)C_6]$$
$$+ (\kappa_\rho/\kappa)[-(6^{1/2}/3)C_4 - C_7 + (3^{1/2}/3)C_8]$$
$$+ k[-(1/3)(1-\nu^2)^{-1}][(\nu\nu'' - \nu')C_5$$
$$+ (\nu\nu' - \nu'')C_7 - (2/3)^{1/2}\gamma C_{13}]$$

$$0 = -wC_2 + (\kappa_r/\kappa)[-(2^{1/2}/6)C_3 + (3^{1/2}/2)C_5 + (1/6)C_6]$$
$$+ (\kappa_\rho/\kappa)[(2^{1/2}/3)C_4 + (3^{1/2}/3)C_7 - (1/3)C_8]$$
$$+ k[4(2^{1/2}/9)(1-\nu^2)^{-1}][(\nu\nu'' - \nu')C_3$$
$$+ (\nu\nu' - \nu'')C_4 - (2/3)^{1/2}\gamma C_9]$$
$$+ k[(5/9)(1-\nu^2)^{-1}][(\nu\nu'' - \nu')C_6$$
$$+ (\nu\nu' - \nu'')C_8 - (2/3)^{1/2}\gamma C_{14}]$$

$$0 = -wC_3 + (\kappa_r/\kappa)[-(6^{1/2}/2)C_1 - (2^{1/2}/6)C_2 - (1/3)C_{10}]$$
$$+ (\kappa_\rho/\kappa)[-5(6^{1/2}/9)C_9 + (30^{1/2}/9)C_{12} + (1/3)C_{13}$$
$$- (3^{1/2}/9)C_{14} - (6^{1/2}/3)\nu C_1 + (2^{1/2}/3)\nu C_2]$$
$$+ k[-4(2^{1/2}/9)\nu' C_2] + k(1-\nu^2)^{-1}$$
$$\times \{5(6^{1/2}/54)[(3/2)^{1/2}\gamma(\nu C_3 - C_4) + (\nu\nu' - \nu'')C_9]$$
$$+ (30^{1/2}/54)[(6/5)^{1/2}(-\nu\nu'' + \nu')C_{10}$$
$$+ (-\nu\nu' + \nu'')C_{12} - (3/5)^{1/2}\gamma C_{15}]$$
$$-4(3^{1/2}/27)[(3/2)^{1/2}\gamma(\nu C_6 - C_8) + (\nu\nu' - \nu'')C_{14}]\}$$

$$0 = -wC_4 + (\kappa_r/\kappa)[-5(6^{1/2}/18)C_9 - (30^{1/2}/18)C_{12} - (1/2)C_{13}$$
$$-(3^{1/2}/18)C_{14} - (6^{1/2}/2)\nu C_1 - (2^{1/2}/6)\nu C_2]$$
$$+(\kappa_\rho/\kappa)[-(6^{1/2}/3)C_1 + (2^{1/2}/3)C_2 + (2/3)C_{11}]$$
$$+k[-4(2^{1/2}/9)\nu'' C_2] + k(1-\nu^2)^{-1}$$
$$\times\{5(6^{1/2}/54)[(3/2)^{1/2}\gamma(C_3 - \nu C_4) - (\nu\nu'' - \nu')C_9]$$
$$+(30^{1/2}/54)[(6/5)^{1/2}(-\nu\nu' + \nu'')C_{11}$$
$$+(-\nu\nu'' + \nu')C_{12} - (3/5)^{1/2}\gamma C_{16}]$$
$$-4(3^{1/2}/27)[(3/2)^{1/2}\gamma(C_6 - \nu C_8) - (\nu\nu'' - \nu')C_{14}]\}$$

$$0 = -wC_5 + (\kappa_r/\kappa)[(1/2)C_1 + (3^{1/2}/2)C_2 + (6^{1/2}/2)C_{10}]$$
$$+(\kappa_\rho/\kappa)[(1/3)C_9 + (5^{1/2}/3)C_{12} - (6^{1/2}/3)C_{13}$$
$$+(2^{1/2}/3)C_{14} - \nu C_1 + (3^{1/2}/3)\nu C_2]$$
$$+k(1/3)\nu' C_1 + k[-(6^{1/2}/9)(1-\nu^2)^{-1}]$$
$$\times[(3/2)^{1/2}\gamma(\nu C_5 - C_7) + (\nu\nu' - \nu'')C_{13}]$$

$$0 = -wC_6 + (\kappa_r/\kappa)[(3^{1/2}/2)C_1 + (1/6)C_2 + (2^{1/2}/6)C_{10}]$$
$$+(\kappa_\rho/\kappa)[-(3^{1/2}/9)C_9 - (15^{1/2}/9)C_{12} + (2^{1/2}/3)C_{13}$$
$$-(6^{1/2}/9)C_{14} + (3^{1/2}/3)\nu C_1 - (1/3)\nu C_2]$$
$$+k[-(5/9)\nu' C_2] + k(1-\nu^2)^{-1}$$
$$\times\{-4(3^{1/2}/27)[(3/2)^{1/2}\gamma(\nu C_3 - C_4) + (\nu\nu' - \nu'')C_9]$$
$$+4(15^{1/2}/27)[(6/5)^{1/2}(-\nu\nu'' + \nu')C_{10}$$
$$+(-\nu\nu' + \nu'')C_{12} - (3/5)^{1/2}\gamma C_{15}]$$
$$+5(6^{1/2}/27)[(3/2)^{1/2}\gamma(\nu C_6 - C_8) + (\nu\nu' - \nu'')C_{14}]\}$$

$$0 = - wC_7 + (\kappa_r/\kappa)[-(1/2)C_9 + (5^{1/2}/2)C_{12} - (6^{1/2}/6)C_{13}$$
$$- (2^{1/2}/2)C_{14} + (1/2)\nu C_1 + (3^{1/2}/2)\nu C_2]$$
$$+ (\kappa_\rho/\kappa)[-C_1 + (3^{1/2}/3)C_2 + (6^{1/2}/3)C_{11}]$$
$$+ k(1/3)\nu'' C_1 + k[-(6^{1/2}/9)(1-\nu^2)^{-1}]$$
$$\times [(3/2)^{1/2}\gamma(C_5 - \nu C_7) - (\nu\nu'' - \nu')C_{13}]$$

$$0 = - wC_8 + (\kappa_r/\kappa)[-(3^{1/2}/18)C_9 + (15^{1/2}/18)C_{12} - (2^{1/2}/2)C_{13}$$
$$- (6^{1/2}/18)C_{14} + (3^{1/2}/2)\nu C_1 + (1/6)\nu C_2]$$
$$+ (\kappa_\rho/\kappa)[(3^{1/2}/3)C_1 - (1/3)C_2 - (2^{1/2}/3)C_{11}]$$
$$+ k[-(5/9)\nu'' C_2] + k(1-\nu^2)^{-1}$$
$$\times \{-4(3^{1/2}/27)[(3/2)^{1/2}\gamma(C_3 - \nu C_4) - (\nu\nu'' - \nu')C_9]$$
$$+ 4(15^{1/2}/27)[(6/5)^{1/2}(-\nu\nu' + \nu'')C_{11}$$
$$+ (-\nu\nu'' + \nu')C_{12} - (3/5)^{1/2}\gamma C_{16}]$$
$$+ 5(6^{1/2}/27)[(3/2)^{1/2}\gamma(C_6 - \nu C_8) - (\nu\nu'' - \nu')C_{14}]\}$$

$$0 = - wC_9 + (\kappa_r/\kappa)[5(6^{1/2}/12)(\nu C_3 - C_4) + (3/4)(\nu C_5 - C_7)$$
$$+ (3^{1/2}/12)(\nu C_6 - C_8) + (3^{1/2}/6)C_{15}]$$
$$+ (\kappa_\rho/\kappa)[-5(6^{1/2}/6)(C_3 - \nu C_4) + (1/2)(C_5 - \nu C_7)$$
$$- (3^{1/2}/6)(C_6 - \nu C_8) - (3^{1/2}/3)C_{16}]$$
$$+ k[4(3^{1/2}/9)\gamma C_2 + 5(6^{1/2}/36)(-\nu'' C_3 + \nu' C_4)$$
$$+ 2(3^{1/2}/9)(\nu'' C_6 - \nu' C_8)] + k(1-\nu^2)^{-1}$$
$$\times \{(6^{1/2}/18)\gamma(C_{10} + C_{11}) - (5^{1/2}/9)\gamma\nu C_{12}$$
$$+ (3^{1/2}/18)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}$$

$$
\begin{aligned}
0 = &- wC_{10} + (\kappa_r/\kappa)\left[-(1/3)C_3 + (6^{1/2}/2)C_5 + (2^{1/2}/6)C_6\right] \\
&+ (\kappa_\rho/\kappa)\left[(1/3)(3\nu C_3 - C_4) + (6^{1/2}/6)(3\nu C_5 - C_7)\right. \\
&\left.- (2^{1/2}/6)(3\nu C_6 - C_8) + (2^{1/2})C_{15}\right] \\
&+ k\left[(1/6)\nu' C_3 + 2(2^{1/2}/3)\nu' C_6\right] + k(1-\nu^2)^{-1} \\
&\times \left\{(1/18)\left[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\right]\right. \\
&+ 2(2^{1/2}/9)\left[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\right] \\
&- (6^{1/2}/54)\gamma C_9 + (1/3)\gamma\nu C_{10} - (30^{1/2}/18)\gamma C_{12} \\
&\left.- 4(3^{1/2}/27)\gamma C_{14} - (2^{1/2}/6)(\nu\nu' - \nu'')C_{15}\right\} \\
0 = &- wC_{11} + (\kappa_r/\kappa)\left[(1/6)(C_3 - 3\nu C_4) - (6^{1/2}/4)(C_5 - 3\nu C_7)\right. \\
&\left.- (2^{1/2}/12)(C_6 - 3\nu C_8) + (2^{1/2}/2)C_{16}\right] \\
&+ (\kappa_\rho/\kappa)\left[(2/3)C_4 + (6^{1/2}/3)C_7 - (2^{1/2}/3)C_8\right] \\
&+ k\left[(1/6)\nu'' C_4 + 2(2^{1/2}/3)\nu'' C_8\right] + k(1-\nu^2)^{-1} \\
&\times \left\{(1/18)\left[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\right]\right. \\
&+ 2(2^{1/2}/9)\left[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\right] \\
&- (6^{1/2}/54)\gamma C_9 - (1/3)\gamma\nu C_{11} + (30^{1/2}/18)\gamma C_{12} \\
&\left.- 4(3^{1/2}/27)\gamma C_{14} + (2^{1/2}/6)(\nu\nu'' - \nu')C_{16}\right\} \\
0 = &- wC_{12} + (\kappa_r/\kappa)\left[-(30^{1/2}/60)(\nu C_3 + 3C_4) + 3(5^{1/2}/20)(\nu C_5 + 3C_7)\right. \\
&\left.+ (15^{1/2}/60)(\nu C_6 + 3C_8) + (15^{1/2}/10)C_{15}\right] \\
&+ (\kappa_\rho/\kappa)\left[(30^{1/2}/30)(3C_3 + \nu C_4) + (5^{1/2}/10)(3C_5 + \nu C_7)\right. \\
&\left.- (15^{1/2}/30)(3C_6 + \nu C_8) + (15^{1/2}/5)C_{16}\right] \\
&+ k\left[(30^{1/2}/60)(\nu'' C_3 + \nu' C_4) + 2(15^{1/2}/15)(\nu'' C_6 + \nu' C_8)\right] \\
&+ k(1-\nu^2)^{-1}\left\{(30^{1/2}/90)\nu\left[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\right]\right. \\
&+ 4(15^{1/2}/45)\nu\left[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\right] \\
&- (5^{1/2}/45)\gamma\nu C_9 + (30^{1/2}/30)\gamma(C_{10} - C_{11}) \\
&- 4(10^{1/2}/45)\gamma\nu C_{14} \\
&\left.+ (15^{1/2}/30)\left[(\nu\nu'' - \nu')C_{15} - (\nu\nu' - \nu'')C_{16}\right]\right\}
\end{aligned}
$$

$$0 = -wC_{13} + (\kappa_r/\kappa)[(3/4)(\nu C_3 - C_4) + (6^{1/2}/4)(\nu C_5 - C_7)$$
$$+ 3(2^{1/2}/4)(\nu C_6 - C_8) - 3(2^{1/2}/4)C_{15}]$$
$$+ (\kappa_\rho/\kappa)[(1/2)(C_3 - \nu C_4) - (6^{1/2}/2)(C_5 - \nu C_7)$$
$$+ (2^{1/2}/2)(C_6 - \nu C_8) - (2^{1/2}/2)C_{16}]$$
$$+ k[(6^{1/2}/6)(-\gamma C_1 + \nu'' C_5 - \nu' C_7)]$$

$$0 = -w\, W_{14} + (\kappa_r/\kappa)[(3^{1/2}/12)(\nu C_3 - C_4) + 3(2^{1/2}/4)(\nu C_5 - C_7)$$
$$+ (6^{1/2}/12)(\nu C_6 - C_8) - (6^{1/2}/12)C_{15}]$$
$$+ (\kappa_\rho/\kappa)[-(3^{1/2}/6)(C_3 - \nu C_4) + (2^{1/2}/2)(C_5 - \nu C_7)$$
$$- (6^{1/2}/6)(C_6 - \nu C_8) + (6^{1/2}/6)C_{16}]$$
$$+ k[2(3^{1/2}/9)(\nu'' C_3 - \nu' C_4) + 5(6^{1/2}/18)(\gamma C_2 - \nu'' C_6 + \nu' C_8)]$$
$$+ k(1-\nu^2)^{-1}\{4(3^{1/2}/9)\gamma(C_{10} + C_{11}) - 4(10^{1/2}/9)\gamma\nu C_{12}$$
$$+ 2(6^{1/2}/9)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}$$

$$0 = -wC_{15} + (\kappa_r/\kappa)[(3^{1/2}/6)C_9 - (2^{1/2}/2)\nu C_{10}$$
$$+ (15^{1/2}/6)C_{12} - 3(2^{1/2}/4)C_{13} - (6^{1/2}/12)C_{14}]$$
$$+ (\kappa_\rho/\kappa)[-(3^{1/2}/3)\nu C_9 + (2^{1/2})(2C_{10} + C_{11}) - (15^{1/2})\nu C_{12}$$
$$- (2^{1/2}/2)\nu C_{13} + (6^{1/2}/6)\nu C_{14}]$$
$$+ k[(2^{1/2}/12)\gamma C_3 + (2/3)\gamma C_6 - (3^{1/2}/18)\nu' C_9$$
$$+ (2^{1/2}/3)\nu'' C_{10} - (15^{1/2}/9)\nu' C_{12} - 2(6^{1/2}/9)\nu' C_{14}]$$
$$+ k(1-\nu^2)^{-1}\{(2^{1/2}/6)[\nu(\nu\nu'' - \nu')C_{10} - (\nu\nu' - \nu'')C_{11}]$$
$$+ (15^{1/2}/18)[\nu(\nu\nu' - \nu'') - (\nu\nu'' - \nu')]C_{12}$$
$$+ (1/6)\gamma(\nu C_{15} - C_{16})\}$$

$$
\begin{aligned}
0 = & - wC_{16} + (\kappa_r/\kappa)\big[(3^{1/2}/6)\nu C_9 + (2^{1/2}/2)(C_{10} + 2C_{11}) \\
& - (15^{1/2}/2)\nu C_{12} - 3(2^{1/2}/4)\nu C_{13} - (6^{1/2}/12)\nu C_{14}\big] \\
& + (\kappa_\rho/\kappa)\big[-(3^{1/2}/3)C_9 - (2^{1/2})\nu C_{11} + (15^{1/2}/3)C_{12} \\
& - (2^{1/2}/2)C_{13} + (6^{1/2}/6)C_{14}\big] \\
& + k\big[(2^{1/2}/12)\gamma C_4 + (2/3)\gamma C_8 - (3^{1/2}/18)\nu'' C_9 \\
& - (2^{1/2}/3)\nu' C_{11} + (15^{1/2}/9)\nu'' C_{12} - 2(6^{1/2}/9)\nu'' C_{14}\big] \\
& + k(1-\nu^2)^{-1}\big\{(2^{1/2}/6)\big[(\nu\nu'' - \nu')C_{10} - \nu(\nu\nu' - \nu'')C_{11}\big] \\
& + (15^{1/2}/18)\big[-\nu(\nu\nu'' - \nu') + (\nu\nu' - \nu'')\big]C_{12} \\
& + (1/6)\gamma(C_{15} - \nu C_{16})\big\}
\end{aligned}
$$

APPENDIX C: 16 ROWS OF THE H_{tr} MATRIX

$$
\begin{aligned}
H_{tr}\varphi_1 = & (\kappa_r/\kappa)\big[-(6^{1/2}/2)\varphi_3 - (1/2)\varphi_5 + (3^{1/2}/2)\varphi_6\big] \\
& + (\kappa_\rho/\kappa)\big[(6^{1/2}/3)\varphi_4 + \varphi_7 - (3^{1/2}/3)\varphi_8\big] \\
& + k\big[-(6^{1/2}/3)\varphi_3^{0,0,k} + (1/3)\varphi_5^{0,0,k} + (3^{1/2}/3)\varphi_6^{0,0,k}\big] \\
H_{tr}\varphi_2 = & (\kappa_r/\kappa)\big[7(2^{1/2}/6)\varphi_3 + (3^{1/2}/2)\varphi_5 + (11/6)\varphi_6\big] \\
& + (\kappa_\rho/\kappa)\big[-(2^{1/2}/3)\varphi_4 - (3^{1/2}/3)\varphi_7 + (1/3)\varphi_8\big] \\
& + k\big[-(2^{1/2}/9)\varphi_3^{0,0,k} + (3^{1/2}/3)\varphi_5^{0,0,k} + (1/9)\varphi_6^{0,0,k}\big] \\
H_{tr}\varphi_3 = & (\kappa_r/\kappa)\big[-(6^{1/2}/2)\varphi_1 + 7(2^{1/2}/6)\varphi_2 - (2/3)\varphi_{10}\big] \\
& + (\kappa_\rho/\kappa)\big[5(6^{1/2}/9)\varphi_9 - (30^{1/2}/9)\varphi_{12} - (1/3)\varphi_{13} \\
& + (3^{1/2}/9)\varphi_{14} - (2^{1/2}/3)\varphi_{17} + (6^{1/2}/9)\varphi_{18}\big] \\
& + k\big[-(2^{1/2}/3)\varphi_1^{1,0,k} - (6^{1/2}/27)\varphi_2^{1,0,k} + 5(6^{1/2}/27)\varphi_9^{1,0,k} \\
& + (30^{1/2}/27)\varphi_{10}^{1,0,k} + (1/3)\varphi_{13}^{1,0,k} + (3^{1/2}/27)\varphi_{14}^{1,0,k}\big]
\end{aligned}
$$

$$H_{tr}\varphi_4 = (\kappa_r/\kappa)[-5(6^{1/2}/9)\varphi_9 - (30^{1/2}/9)\varphi_{12} - (1/2)\varphi_{13}$$

$$+7(3^{1/2}/18)\varphi_{14} + (2^{1/2}/2)\varphi_{17} - 7(6^{1/2}/18)\varphi_{18}]$$

$$+(\kappa_\rho/\kappa)[(6^{1/2}/3)\varphi_1 - (2^{1/2}/3)\varphi_2 - (2/3)\varphi_{11}]$$

$$+k[-(2^{1/2}/3)\varphi_1^{0,1,k} - (6^{1/2}/27)\varphi_2^{0,1,k} + 5(6^{1/2}/27)\varphi_9^{0,1,k}$$

$$+(30^{1/2}/27)\varphi_{10}^{0,1,k} + (1/3)\varphi_{13}^{0,1,k} + (3^{1/2}/27)\varphi_{14}^{0,1,k}]$$

$$H_{tr}\varphi_5 \doteq (\kappa_r/\kappa)[-(1/2)\varphi_1 + (3^{1/2}/2)\varphi_2 + (6^{1/2}/2)\varphi_{10}]$$

$$+(\kappa_\rho/\kappa)[-(1/3)\varphi_9 - (5^{1/2}/3)\varphi_{12} + (6^{1/2}/3)\varphi_{13}$$

$$-(2^{1/2}/3)\varphi_{14} - (3^{1/2}/3)\varphi_{17} + (1/3)\varphi_{18}]$$

$$+k[(3^{1/2}/9)\varphi_1^{1,0,k} + (1/3)\varphi_2^{1,0,k} + (1/3)\varphi_9^{1,0,k}$$

$$-(5^{1/2}/3)\varphi_{10}^{1,0,k} + (6^{1/2}/9)\varphi_{13}^{1,0,k} + (2^{1/2}/3)\varphi_{14}^{1,0,k}]$$

$$H_{tr}\varphi_6 = (\kappa_r/\kappa)[(3^{1/2}/2)\varphi_1 + (11/6)\varphi_2 - 7(2^{1/2}/6)\varphi_{10}]$$

$$+(\kappa_\rho/\kappa)[(3^{1/2}/9)\varphi_9 + (15^{1/2}/9)\varphi_{12} - (2^{1/2}/3)\varphi_{13}$$

$$+(6^{1/2}/9)\varphi_{14} + (1/3)\varphi_{17} - (3^{1/2}/9)\varphi_{18}]$$

$$+k[(1/3)\varphi_1^{1,0,k} + (3^{1/2}/27)\varphi_2^{1,0,k} + (3^{1/2}/27)\varphi_9^{1,0,k}$$

$$-(15^{1/2}/27)\varphi_{10}^{1,0,k} + (2^{1/2}/3)\varphi_{13}^{1,0,k} + (6^{1/2}/27)\varphi_{14}^{1,0,k}]$$

$$H_{tr}\varphi_7 = (\kappa_r/\kappa)[-(1/2)\varphi_9 + (5^{1/2}/2)\varphi_{12} + (6^{1/2}/6)\varphi_{13}$$

$$-(2^{1/2}/2)\varphi_{14} + (3^{1/2}/6)\varphi_{17} - (1/2)\varphi_{18}]$$

$$+(\kappa_\rho/\kappa)[\varphi_1 - (3^{1/2}/3)\varphi_2 - (6^{1/2}/3)\varphi_{11}]$$

$$+k[(3^{1/2}/9)\varphi_1^{0,1,k} + (1/3)\varphi_2^{0,1,k} + (1/3)\varphi_9^{0,1,k}$$

$$-(5^{1/2}/3)\varphi_{10}^{0,1,k} + (6^{1/2}/9)\varphi_{13}^{0,1,k} + (2^{1/2}/3)\varphi_{14}^{0,1,k}]$$

$$H_{tr}\varphi_8 = (\kappa_r/\kappa)[7(3^{1/2}/18)\varphi_9 - 7(15^{1/2}/18)\varphi_{12} - (2^{1/2}/2)\varphi_{13}$$
$$- 11(6^{1/2}/18)\varphi_{14} - (1/2)\varphi_{17} - 11(3^{1/2}/18)\varphi_{18}]$$
$$+ (\kappa_\rho/\kappa)[-(3^{1/2}/3)\varphi_1 + (1/3)\varphi_2 + (2^{1/2}/3)\varphi_{11}]$$
$$+ k[(1/3)\varphi_1^{0,1,k} + (3^{1/2}/27)\varphi_2^{0,1,k} + (3^{1/2}/27)\varphi_9^{0,1,k}$$
$$- (15^{1/2}/27)\varphi_{10}^{0,1,k} + (2^{1/2}/3)\varphi_{13}^{0,1,k} + (6^{1/2}/27)\varphi_{14}^{0,1,k}]$$

$$H_{tr}\varphi_9 = (\kappa_r/\kappa)[-5(6^{1/2}/9)\varphi_4 - (1/2)\varphi_7 + 7(3^{1/2}/18)\varphi_8$$
$$+ (3^{1/2}/3)\varphi_{15} - 5(3^{1/2}/9)\varphi_{19} - (2^{1/2}/4)\varphi_{21}$$
$$+ 7(6^{1/2}/36)\varphi_{22}]$$
$$+ (\kappa_\rho/\kappa)[5(6^{1/2}/9)\varphi_3 - (1/3)\varphi_5 + (3^{1/2}/9)\varphi_6 + (3^{1/2}/3)\varphi_{16}$$
$$+ 5(3^{1/2}/9)\varphi_{20} - (2^{1/2}/6)\varphi_{23} + (6^{1/2}/18)\varphi_{24}]$$
$$+ k[-(2^{1/2}/3)\varphi_1^{1,1,k} - (6^{1/2}/27)\varphi_2^{1,1,k} - 5(6^{1/2}/27)\varphi_{3b}^{1,1,k}$$
$$- (1/3)\varphi_{5b}^{1,1,k} - (3^{1/2}/27)\varphi_{6b}^{1,1,k} + (30^{1/2}/27)\varphi_{15a}^{1,1,k}]$$

$$H_{tr}\varphi_{10} = (\kappa_r/\kappa)[-(2/3)\varphi_3 + (6^{1/2}/2)\varphi_5 - 7(2^{1/2}/6)\varphi_6]$$
$$+ (\kappa_\rho/\kappa)[-(2^{1/2})\varphi_{15} + (2^{1/2}/3)\varphi_{19} + (3^{1/2}/3)\varphi_{21} - (1/3)\varphi_{22}]$$
$$+ k[(2^{1/2}/9)\varphi_3^{2,0,k} - (3^{1/2}/3)\varphi_5^{2,0,k} - (1/9)\varphi_6^{2,0,k} + (2^{1/2}/3)\varphi_{15}^{2,0,k}]$$

$$H_{tr}\varphi_{11} = (\kappa_r/\kappa)[(2^{1/2})\varphi_{16} + (2^{1/2}/3)\varphi_{20} - (3^{1/2}/2)\varphi_{23} + (7/6)\varphi_{24}]$$
$$+ (\kappa_\rho/\kappa)[-(2/3)\varphi_4 - (6^{1/2}/3)\varphi_7 + (2^{1/2}/3)\varphi_8]$$
$$+ k[(2^{1/2}/9)\varphi_3^{0,2,k} - (3^{1/2}/3)\varphi_5^{0,2,k} - (1/9)\varphi_6^{0,2,k} + (2^{1/2}/3)\varphi_{15}^{0,2,k}]$$

$$H_{tr}\varphi_{12} = (\kappa_r/\kappa)[-(30^{1/2}/9)\varphi_4 + (5^{1/2}/2)\varphi_7 - 7(15^{1/2}/18)\varphi_8$$
$$+ (15^{1/2}/5)\varphi_{15} + (15^{1/2}/45)\varphi_{19} - (10^{1/2}/20)\varphi_{21} + 7(30^{1/2}/180)\varphi_{22}]$$
$$+ (\kappa_\rho/\kappa)[-(30^{1/2}/9)\varphi_3 - (5^{1/2}/3)\varphi_5 + (15^{1/2}/9)\varphi_6 - (15^{1/2}/5)\varphi_{16}$$
$$+ (15^{1/2}/45)\varphi_{20} + (10^{1/2}/30)\varphi_{23} - (30^{1/2}/90)\varphi_{24}]$$
$$+ k[2(2^{1/2}/27)\varphi_{3s}^{1,1,k} + (10^{1/2}/27)\varphi_{3c}^{1,1,k} - 2(3^{1/2}/9)\varphi_{5s}^{1,1,k}$$
$$- (15^{1/2}/9)\varphi_{5c}^{1,1,k} - (2/27)\varphi_{6s}^{1,1,k} - (5^{1/2}/27)\varphi_{6c}^{1,1,k} + (2^{1/2}/3)\varphi_{15s}^{1,1,k}]$$

$$H_{tr}\varphi_{13} = (\kappa_r/\kappa)[-(1/2)\varphi_4 + (6^{1/2}/6)\varphi_7 - (2^{1/2}/2)\varphi_8$$

$$-3(2^{1/2}/4)\varphi_{15} - (2^{1/2}/4)\varphi_{19} + (3^{1/2}/6)\varphi_{21} - (1/2)\varphi_{22}]$$

$$+(\kappa_\rho/\kappa)[-(1/3)\varphi_3 + (6^{1/2}/3)\varphi_5 - (2^{1/2}/3)\varphi_6 + (2^{1/2}/2)\varphi_{16}$$

$$-(2^{1/2}/6)\varphi_{20} + (3^{1/2}/3)\varphi_{23} - (1/3)\varphi_{24}]$$

$$+k[(3^{1/2}/9)\varphi_1^{1,1,k} + (1/3)\varphi_2^{1,1,k} - (1/3)\varphi_{3b}^{1,1,k}$$

$$-(6^{1/2}/9)\varphi_{5b}^{1,1,k} - (2^{1/2}/3)\varphi_{6b}^{1,1,k} - (5^{1/2}/3)\varphi_{15a}^{1,1,k}]$$

$$H_{tr}\varphi_{14} = (\kappa_r/\kappa)[7(3^{1/2}/18)\varphi_4 - (2^{1/2}/2)\varphi_7 - 11(6^{1/2}/18)\varphi_8$$

$$+7(6^{1/2}/12)\varphi_{15} + 7(6^{1/2}/36)\varphi_{19} - (1/2)\varphi_{21}$$

$$-11(3^{1/2}/18)\varphi_{22}]$$

$$+(\kappa_\rho/\kappa)[(3^{1/2}/9)\varphi_3 - (2^{1/2}/3)\varphi_5 + (6^{1/2}/9)\varphi_6 - (6^{1/2}/6)\varphi_{16}$$

$$+(6^{1/2}/18)\varphi_{20} - (1/3)\varphi_{23} + (3^{1/2}/9)\varphi_{24}]$$

$$+k[(1/3)\varphi_1^{1,1,k} + (3^{1/2}/27)\varphi_2^{1,1,k} - (3^{1/2}/27)\varphi_{3b}^{1,1,k}$$

$$-(2^{1/2}/3)\varphi_{5b}^{1,1,k} - (6^{1/2}/27)\varphi_{6b}^{1,1,k} - (15^{1/2}/27)\varphi_{15a}^{1,1,k}]$$

$$H_{tr}\varphi_{15} = (\kappa_r/\kappa)[(3^{1/2}/3)\varphi_9 + (15^{1/2}/5)\varphi_{12} - 3(2^{1/2}/4)\varphi_{13}$$

$$+7(6^{1/2}/12)\varphi_{14} + (10^{1/2}/5)\varphi_{26}]$$

$$+(\kappa_\rho/\kappa)[-(2^{1/2})\varphi_{10} - (15^{1/2}/15)\varphi_{25} - (35^{1/2}/5)\varphi_{28}$$

$$-(10^{1/2}/10)\varphi_{29} + (30^{1/2}/30)\varphi_{30}]$$

$$+k[(2^{1/2}/9)\varphi_{9a}^{2,1,k} - (2^{1/2}/3)\varphi_{10b}^{2,1,k} - (3^{1/2}/3)\varphi_{13a}^{2,1,k}$$

$$-(1/9)\varphi_{14a}^{2,1,k}]$$

$$H_{tr}\varphi_{16} = (\kappa_r/\kappa)[(2^{1/2})\varphi_{11} - (15^{1/2}/15)\varphi_{25} + (35^{1/2}/5)\varphi_{28}$$
$$+ 3(10^{1/2}/20)\varphi_{29} - 7(30^{1/2}/60)\varphi_{30}]$$
$$+ (\kappa_\rho/\kappa)[(3^{1/2}/3)\varphi_9 - (15^{1/2}/5)\varphi_{12} + (2^{1/2}/2)\varphi_{13}$$
$$- (6^{1/2}/6)\varphi_{14} - (10^{1/2}/5)\varphi_{27}]$$
$$+ k[(2^{1/2}/9)\varphi_{9a}^{1,2,k} + (2^{1/2}/3)\varphi_{10b}^{1,2,k} - (3^{1/2}/3)\varphi_{13}^{1,2,k}$$
$$- (1/9)\varphi_{14a}^{1,2,k}]$$

APPENDIX D: 16 ROWS OF THE $H_{t\rho}$ MATRIX

$$H_{t\rho}\varphi_1 = (\kappa_r/\kappa)[(6^{1/2}/6)\varphi_3 + (1/2)\varphi_5 - (3^{1/2}/6)\varphi_6]$$
$$+ (\kappa_\rho/\kappa)[(6^{1/2}/3)\varphi_4 - \varphi_7 - (3^{1/2}/3)\varphi_8]$$
$$+ k[-(6^{1/2}/9)\varphi_3^{0,0,k} - (1/3)\varphi_5^{0,0,k} + (3^{1/2}/9)\varphi_6^{0,0,k}]$$
$$H_{t\rho}\varphi_2 = (\kappa_r/\kappa)[-(2^{1/2}/6)\varphi_3 - (3^{1/2}/6)\varphi_5 + (1/6)\varphi_6]$$
$$+ (\kappa_\rho/\kappa)[(2^{1/2})\varphi_4 - (3^{1/2}/3)\varphi_7 + \varphi_8]$$
$$+ k[(2^{1/2}/9)\varphi_3^{0,0,k} + (3^{1/2}/9)\varphi_5^{0,0,k} - (1/9)\varphi_6^{0,0,k}]$$
$$H_{t\rho}\varphi_3 = (\kappa_r/\kappa)[(6^{1/2}/6)\varphi_1 - (2^{1/2}/6)\varphi_2 - (1/3)\varphi_{10}]$$
$$+ (\kappa_\rho/\kappa)[-(1/3)\varphi_{13} - (3^{1/2}/3)\varphi_{14} - (2^{1/2}/3)\varphi_{17} - (6^{1/2}/3)\varphi_{18}]$$
$$+ k[-(2^{1/2}/9)\varphi_1^{1,0,k} + (6^{1/2}/27)\varphi_2^{1,0,k} - 5(6^{1/2}/27)\varphi_9^{1,0,k}$$
$$- (30^{1/2}/27)\varphi_{10}^{1,0,k} + (1/9)\varphi_{13}^{1,0,k} - (3^{1/2}/27)\varphi_{14}^{1,0,k}]$$
$$H_{t\rho}\varphi_4 = (\kappa_r/\kappa)[-5(6^{1/2}/18)\varphi_9 - (30^{1/2}/18)\varphi_{12} + (1/6)\varphi_{13}$$
$$- (3^{1/2}/18)\varphi_{14} - (2^{1/2}/6)\varphi_{17} + (6^{1/2}/18)\varphi_{18}]$$
$$+ (\kappa_\rho/\kappa)[(6^{1/2}/3)\varphi_1 + (2^{1/2})\varphi_2]$$
$$+ k[-(2^{1/2}/9)\varphi_1^{0,1,k} + (6^{1/2}/27)\varphi_2^{0,1,k} - 5(6^{1/2}/27)\varphi_9^{0,1,k}$$
$$- (30^{1/2}/27)\varphi_{10}^{0,1,k} + (1/9)\varphi_{13}^{0,1,k} - (3^{1/2}/27)\varphi_{14}^{0,1,k}]$$

$$H_{t\rho}\varphi_5 = (\kappa_r/\kappa)[(1/2)\varphi_1 - (3^{1/2}/6)\varphi_2 - (6^{1/2}/6)\varphi_{10}]$$

$$+ (\kappa_\rho/\kappa)[-(1/3)\varphi_9 - (5^{1/2}/3)\varphi_{12} - (6^{1/2}/3)\varphi_{13}$$

$$- (2^{1/2}/3)\varphi_{14} + (3^{1/2}/3)\varphi_{17} + (1/3)\varphi_{18}]$$

$$+ k[-(3^{1/2}/9)\varphi_1^{1,0,k} + (1/9)\varphi_2^{1,0,k} + (1/9)\varphi_9^{1,0,k}$$

$$- (5^{1/2}/9)\varphi_{10}^{1,0,k} - (6^{1/2}/9)\varphi_{13}^{1,0,k} + (2^{1/2}/9)\varphi_{14}^{1,0,k}]$$

$$H_{t\rho}\varphi_6 = (\kappa_r/\kappa)[-(3^{1/2}/6)\varphi_1 + (1/6)\varphi_2 + (2^{1/2}/6)\varphi_{10}]$$

$$+ (\kappa_\rho/\kappa)[-(3^{1/2}/3)\varphi_9 - (15^{1/2}/3)\varphi_{12} - (2^{1/2}/3)\varphi_{13}$$

$$+ (6^{1/2}/3)\varphi_{14} + (1/3)\varphi_{17} - (3^{1/2}/3)\varphi_{18}]$$

$$+ k[(1/9)\varphi_1^{1,0,k} - (3^{1/2}/27)\varphi_2^{1,0,k} - (3^{1/2}/27)\varphi_9^{1,0,k}$$

$$+ (15^{1/2}/27)\varphi_{10}^{1,0,k} + (2^{1/2}/9)\varphi_{13}^{1,0,k} - (6^{1/2}/27)\varphi_{14}^{1,0,k}]$$

$$H_{t\rho}\varphi_7 = (\kappa_r/\kappa)[(1/6)\varphi_9 - (5^{1/2}/6)\varphi_{12} - (6^{1/2}/6)\varphi_{13}$$

$$+ (2^{1/2}/6)\varphi_{14} - (3^{1/2}/6)\varphi_{17} + (1/6)\varphi_{18}]$$

$$+ (\kappa_\rho/\kappa)[-\varphi_1 - (3^{1/2}/3)\varphi_2 - (6^{1/2}/3)\varphi_{11}]$$

$$+ k[-(3^{1/2}/9)\varphi_1^{0,1,k} + (1/9)\varphi_2^{0,1,k} + (1/9)\varphi_9^{0,1,k}$$

$$- (5^{1/2}/9)\varphi_{10}^{0,1,K} - (6^{1/2}/9)\varphi_{13}^{0,1,k} + (2^{1/2}/9)\varphi_{14}^{0,1,k}]$$

$$H_{t\rho}\varphi_8 = (\kappa_r/\kappa)[-(3^{1/2}/18)\varphi_9 + (15^{1/2}/18)\varphi_{12} + (2^{1/2}/6)\varphi_{13}$$

$$- (6^{1/2}/18)\varphi_{14} + (1/6)\varphi_{17} - (3^{1/2}/18)\varphi_{18}]$$

$$+ (\kappa_\rho/\kappa)[-(3^{1/2}/3)\varphi_1 + \varphi_2 - (2^{1/2})\varphi_{11}]$$

$$+ k[(1/9)\varphi_1^{0,1,k} - (3^{1/2}/27)\varphi_2^{0,1,k} - (3^{1/2}/27)\varphi_9^{0,1,k}$$

$$+ (15^{1/2}/27)\varphi_{10}^{0,1,k} + (2^{1/2}/9)\varphi_{13}^{0,1,k} - (6^{1/2}/27)\varphi_{14}^{0,1,k}]$$

$$H_{t\rho}\varphi_9 = (\kappa_r/\kappa)[-5(6^{1/2}/18)\varphi_4 + (1/6)\varphi_7 - (3^{1/2}/18)\varphi_8$$
$$+ (3^{1/2}/6)\varphi_{15} - 5(3^{1/2}/18)\varphi_{19} + (2^{1/2}/12)\varphi_{21}$$
$$- (6^{1/2}/36)\varphi_{22}]$$
$$+ (\kappa_\rho/\kappa)[-(1/3)\varphi_5 - (3^{1/2}/3)\varphi_6 - (2^{1/2}/6)\varphi_{23} - (6^{1/2}/6)\varphi_{24}]$$
$$+ k[-(2^{1/2}/9)\varphi_1^{1,1,k} + (6^{1/2}/27)\varphi_2^{1,1,k} + 5(6^{1/2}/27)\varphi_{3b}^{1,1,k}$$
$$- (1/9)\varphi_{5b}^{1,1,k} + (3^{1/2}/27)\varphi_{6b}^{1,1,k} - (30^{1/2}/27)\varphi_{15a}^{1,1,k}]$$

$$H_{t\rho}\varphi_{10} = (\kappa_r/\kappa)[-(1/3)\varphi_3 - (6^{1/2}/6)\varphi_5 + (2^{1/2}/6)\varphi_6]$$
$$+ (\kappa_\rho/\kappa)[(3^{1/2}/3)\varphi_{21} + \varphi_{22}]$$
$$+ k[-(2^{1/2}/9)\varphi_3^{2,0,k} - (3^{1/2}/9)\varphi_5^{2,0,k} + (1/9)\varphi_6^{2,0,k}$$
$$- (2^{1/2}/3)\varphi_{15}^{2,0,k}]$$

$$H_{t\rho}\varphi_{11} = (\kappa_r/\kappa)[(2^{1/2}/2)\varphi_{16} + (2^{1/2}/6)\varphi_{20} + (3^{1/2}/6)\varphi_{23} - (1/6)\varphi_{24}]$$
$$+ (\kappa_\rho/\kappa)[-(6^{1/2}/3)\varphi_7 - (2^{1/2})\varphi_8]$$
$$+ k[-(2^{1/2}/9)\varphi_3^{0,2,k} - (3^{1/2}/9)\varphi_5^{0,2,k} + (1/9)\varphi_6^{0,2,k}$$
$$- (2^{1/2}/3)\varphi_{15}^{0,2,k}]$$

$$H_{t\rho}\varphi_{12} = (\kappa_r/\kappa)[-(30^{1/2}/18)\varphi_4 - (5^{1/2}/6)\varphi_7 + (15^{1/2}/18)\varphi_8$$
$$+ (15^{1/2}/10)\varphi_{15} + (15^{1/2}/90)\varphi_{19} + (10^{1/2}/60)\varphi_{21}$$
$$- (30^{1/2}/180)\varphi_{22}]$$
$$+ (\kappa_\rho/\kappa)[-(5^{1/2}/3)\varphi_5 - (15^{1/2}/3)\varphi_6 + (10^{1/2}/30)\varphi_{23}$$
$$+ (30^{1/2}/30)\varphi_{24}]$$
$$+ k[-2(2^{1/2}/27)\varphi_{3s}^{1,1,k} - (10^{1/2}/27)\varphi_{3c}^{1,1,k} - 2(3^{1/2}/27)\varphi_{5s}^{1,1,k}$$
$$- (15^{1/2}/27)\varphi_{5c}^{1,1,k} + (2/27)\varphi_{6s}^{1,1,k} + (5^{1/2}/27)\varphi_{6c}^{1,1,k}$$
$$- (2^{1/2}/3)\varphi_{15s}^{1,1,k}]$$

$$H_{t\rho}\varphi_{13} = (\kappa_r/\kappa)[(1/6)\varphi_4 - (6^{1/2}/6)\varphi_7 + (2^{1/2}/6)\varphi_8$$
$$+ (2^{1/2}/4)\varphi_{15} + (2^{1/2}/12)\varphi_{19} - (3^{1/2}/6)\varphi_{21} + (1/6)\varphi_{22}]$$
$$+ (\kappa_\rho/\kappa)[-(1/3)\varphi_3 - (6^{1/2}/3)\varphi_5 - (2^{1/2}/3)\varphi_6 + (2^{1/2}/2)\varphi_{16}$$
$$-(2^{1/2}/6)\varphi_{20} - (3^{1/2}/3)\varphi_{23} - (1/3)\varphi_{24}]$$
$$+ k[-(3^{1/2}/9)\varphi_1^{1,1,k} + (1/9)\varphi_2^{1,1,k} - (1/9)\varphi_{3b}^{1,1,k}$$
$$+ (6^{1/2}/9)\varphi_{5b}^{1,1,k} - (2^{1/2}/9)\varphi_{6b}^{1,1,k} - (5^{1/2}/9)\varphi_{15a}^{1,1,k}]$$

$$H_{t\rho}\varphi_{14} = (\kappa_r/\kappa)[-(3^{1/2}/18)\varphi_4 + (2^{1/2}/6)\varphi_7 - (6^{1/2}/18)\varphi_8$$
$$-(6^{1/2}/12)\varphi_{15} - (6^{1/2}/36)\varphi_{19} + (1/6)\varphi_{21} - (3^{1/2}/18)\varphi_{22}]$$
$$+ (\kappa_\rho/\kappa)[-(3^{1/2}/3)\varphi_3 - (2^{1/2}/3)\varphi_5 + (6^{1/2}/3)\varphi_6 + (6^{1/2}/2)\varphi_{16}$$
$$-(6^{1/2}/6)\varphi_{20} - (1/3)\varphi_{23} + (3^{1/2}/3)\varphi_{24}]$$
$$+ k[(1/9)\varphi_1^{1,1,k} - (3^{1/2}/27)\varphi_2^{1,1,k} + (3^{1/2}/27)\varphi_{3b}^{1,1,k}$$
$$-(2^{1/2}/9)\varphi_{5b}^{1,1,k} + (6^{1/2}/27)\varphi_{6b}^{1,1,k} + (15^{1/2}/27)\varphi_{15a}^{1,1,k}]$$

$$H_{t\rho}\varphi_{15} = (\kappa_r/\kappa)[(3^{1/2}/6)\varphi_9 + (15^{1/2}/10)\varphi_{12} + (2^{1/2}/4)\varphi_{13}$$
$$-(6^{1/2}/12)\varphi_{14} + (10^{1/2}/10)\varphi_{26}]$$
$$+ (\kappa_\rho/\kappa)[-(10^{1/2}/10)\varphi_{29} - (30^{1/2}/10)\varphi_{30}]$$
$$+ k[-(2^{1/2}/9)\varphi_{9a}^{2,1,k} + (2^{1/2}/3)\varphi_{10b}^{2,1,k} - (3^{1/2}/9)\varphi_{13a}^{2,1,k}$$
$$+ (1/9)\varphi_{14a}^{2,1,k}]$$

$$H_{t\rho}\varphi_{16} = (\kappa_r/\kappa)[(2^{1/2}/2)\varphi_{11} - (15^{1/2}/30)\varphi_{25} + (35^{1/2}/10)\varphi_{28}$$
$$-(10^{1/2}/20)\varphi_{29} + (30^{1/2}/60)\varphi_{30}]$$
$$+ (\kappa_\rho/\kappa)[(2^{1/2}/2)\varphi_{13} + (6^{1/2}/2)\varphi_{14}]$$
$$+ k[-(2^{1/2}/9)\varphi_{9a}^{1,2,k} - (2^{1/2}/3)\varphi_{10b}^{1,2,k} - (3^{1/2}/9)\varphi_{13a}^{1,2,k}$$
$$+ (1/9)\varphi_{14a}^{1,2,k}]$$

APPENDIX E: THE REDUCED RELATIVE-TIME WAVE EQUATIONS

$$
\begin{aligned}
0 = &- w_r C_1 + (\kappa_r/\kappa)\left[-(6^{1/2}/2)C_3 - (1/2)C_5 + (3^{1/2}/2)C_6\right] \\
&+ (\kappa_\rho/\kappa)\left[(6^{1/2}/3)C_4 + C_7 - (3^{1/2}/3)C_8\right] \\
&+ k(1-\nu^2)^{-1}\{-(6^{1/2}/3)\left[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\right] \\
&+ (1/3)\left[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7\right] \\
&+ (3^{1/2}/3)\left[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\right] \\
&+ (2/3)\gamma C_9 - (6^{1/2}/9)\gamma C_{13} - (2^{1/2}/3)\gamma C_{14}\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_2 + (\kappa_r/\kappa)\left[7(2^{1/2}/6)C_3 + (3^{1/2}/2)C_5 + (11/6)C_6\right] \\
&+ (\kappa_\rho/\kappa)\left[-(2^{1/2}/3)C_4 - (3^{1/2}/3)C_7 + (1/3)C_8\right] \\
&+ k(1-\nu^2)^{-1}\{-(2^{1/2}/9)\left[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\right] \\
&+ (3^{1/2}/3)\left[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7\right] \\
&+ (1/9)\left[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\right] \\
&+ 2(3^{1/2}/27)\gamma C_9 - (2^{1/2}/3)\gamma C_{13} - (6^{1/2}/27)\gamma C_{14}\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_3 + (\kappa_r/\kappa)\left[-(6^{1/2}/2)C_1 + 7(2^{1/2}/6)C_2 - (2/3)C_{10}\right] \\
&+ (\kappa_\rho/\kappa)\left[(6^{1/2}/3)\nu C_1 - (2^{1/2}/3)\nu C_2 + 5(6^{1/2}/9)C_9\right. \\
&\left.- (30^{1/2}/9)C_{12} - (1/3)C_{13} + (3^{1/2}/9)C_{14}\right] \\
&+ k\left[(6^{1/2}/3)\nu' C_1 + (2^{1/2}/9)\nu' C_2\right] \\
&+ k(1-\nu^2)^{-1}\{(5/9)\gamma(\nu C_3 - C_4) + (6^{1/2}/6)\gamma(\nu C_5 - C_7) \\
&+ (2^{1/2}/18)\gamma(\nu C_6 - C_8) + 5(6^{1/2}/27)(\nu\nu' - \nu'')C_9 \\
&- (2/9)(\nu\nu'' - \nu')C_{10} - (30^{1/2}/27)(\nu\nu' - \nu'')C_{12} \\
&+ (1/3)(\nu\nu' - \nu'')C_{13} + (3^{1/2}/27)(\nu\nu' - \nu'')C_{14} \\
&- (2^{1/2}/9)\gamma C_{15}\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_4 + (\kappa_r/\kappa)\big[-(6^{1/2}/2)\nu C_1 + 7(2^{1/2}/6)\nu C_2 \\
&- 5(6^{1/2}/9)C_9 - (30^{1/2}/9)C_{12} - (1/2)C_{13} \\
&+ 7(3^{1/2}/18)C_{14}\big] \\
&+ (\kappa_\rho/\kappa)\big[(6^{1/2}/3)C_1 - (2^{1/2}/3)C_2 - (2/3)C_{11}\big] \\
&+ k\big[(6^{1/2}/3)\nu'' C_1 + (2^{1/2}/9)\nu'' C_2\big] \\
&+ k(1-\nu^2)^{-1}\big\{(5/9)\gamma(C_3 - \nu C_4) + (6^{1/2}/6)\gamma(C_5 - \nu C_7) \\
&+ (2^{1/2}/18)\gamma(C_6 - \nu C_8) - 5(6^{1/2}/27)(\nu\nu'' - \nu')C_9 \\
&- (2/9)(\nu\nu' - \nu'')C_{11} - (30^{1/2}/27)(\nu\nu'' - \nu')C_{12} \\
&- (1/3)(\nu\nu'' - \nu')C_{13} - (3^{1/2}/27)(\nu\nu'' - \nu')C_{14} - (2^{1/2}/9)\gamma C_{16}\big\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_5 + (\kappa_r/\kappa)\big[-(1/2)C_1 + (3^{1/2}/2)C_2 + (6^{1/2}/2)C_{10}\big] \\
&+ (\kappa_\rho/\kappa)\big[\nu C_1 - (3^{1/2}/3)\nu C_2 - (1/3)C_9 - (5^{1/2}/3)C_{12} \\
&+ (6^{1/2}/3)C_{13} - (2^{1/2}/3)C_{14}\big] \\
&+ k\big[-(1/3)\nu' C_1 - (3^{1/2}/3)\nu' C_2\big] \\
&+ k(1-\nu^2)^{-1}\big\{(6^{1/2}/6)\gamma(\nu C_3 - C_4) + (1/3)\gamma(\nu C_5 - C_7) \\
&+ (3^{1/2}/3)\gamma(\nu C_6 - C_8) + (1/3)(\nu\nu' - \nu'')C_9 \\
&+ (6^{1/2}/3)(\nu\nu'' - \nu')C_{10} + (5^{1/2}/3)(\nu\nu' - \nu'')C_{12} \\
&+ (6^{1/2}/9)(\nu\nu' - \nu'')C_{13} + (2^{1/2}/3)(\nu\nu' - \nu'')C_{14} \\
&+ (3^{1/2}/3)\gamma C_{15}\big\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_6 + (\kappa_r/\kappa)\big[(3^{1/2}/2)C_1 + (11/6)C_2 - 7(2^{1/2}/6)C_{10}\big] \\
&+ (\kappa_\rho/\kappa)\big[-(3^{1/2}/3)\nu C_1 + (1/3)\nu C_2 + (3^{1/2}/9)C_9 \\
&+ (15^{1/2}/9)C_{12} - (2^{1/2}/3)C_{13} + (6^{1/2}/9)C_{14}\big] \\
&+ k\big[-(3^{1/2}/3)\nu' C_1 - (1/9)\nu' C_2\big] \\
&+ k(1-\nu^2)^{-1}\big\{(2^{1/2}/18)\gamma(\nu C_3 - C_4) + (3^{1/2}/3)\gamma(\nu C_5 - C_7) \\
&+ (1/9)\gamma(\nu C_6 - C_8) + (3^{1/2}/27)(\nu\nu' - \nu'')C_9 \\
&+ (2^{1/2}/9)(\nu\nu'' - \nu')C_{10} + (15^{1/2}/27)(\nu\nu' - \nu'')C_{12} \\
&+ (2^{1/2}/3)(\nu\nu' - \nu'')C_{13} + (6^{1/2}/27)(\nu\nu' - \nu'')C_{14} \\
&+ (1/9)\gamma C_{15}\big\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_7 + (\kappa_r/\kappa)[-(1/2)\nu C_1 + (3^{1/2}/2)\nu C_2 - (1/2)C_9 \\
&+ (5^{1/2}/2)C_{12} + (6^{1/2}/6)C_{13} - (2^{1/2}/2)C_{14}] \\
&+ (\kappa_\rho/\kappa)[C_1 - (3^{1/2}/3)C_2 - (6^{1/2}/3)C_{11}] \\
&+ k[-(1/3)\nu'' C_1 - (3^{1/2}/3)\nu'' C_2] \\
&+ k(1-\nu^2)^{-1}\{(6^{1/2}/6)\gamma(C_3 - \nu C_4) + (1/3)\gamma(C_5 - \nu C_7) \\
&+ (3^{1/2}/3)\gamma(C_6 - \nu C_8) - (1/3)(\nu\nu'' - \nu')C_9 \\
&+ (6^{1/2}/3)(\nu\nu' - \nu'')C_{11} + (5^{1/2}/3)(\nu\nu'' - \nu')C_{12} \\
&- (6^{1/2}/9)(\nu\nu'' - \nu')C_{13} - (2^{1/2}/3)(\nu\nu'' - \nu')C_{14} \\
&+ (3^{1/2}/3)\gamma C_{16}\} \\
0 = &- w_r C_8 + (\kappa_r/\kappa)[(3^{1/2}/2)\nu C_1 + (11/6)\nu C_2 \\
&+ 7(3^{1/2}/18)C_9 - 7(15^{1/2}/18)C_{12} - (2^{1/2}/2)C_{13} \\
&- 11(6^{1/2}/18)C_{14}] \\
&+ (\kappa_\rho/\kappa)[-(3^{1/2}/3)C_1 + (1/3)C_2 + (2^{1/2}/3)C_{11}] \\
&+ k[-(3^{1/2}/3)\nu'' C_1 - (1/9)\nu'' C_2] \\
&+ k(1-\nu^2)^{-1}\{(2^{1/2}/18)\gamma(C_3 - \nu C_4) \\
&+ (3^{1/2}/3)\gamma(C_5 - \nu C_7) + (1/9)\gamma(C_6 - \nu C_8) \\
&- (3^{1/2}/27)(\nu\nu'' - \nu')C_9 + (2^{1/2}/9)(\nu\nu' - \nu'')C_{11} \\
&+ (15^{1/2}/27)(\nu\nu'' - \nu')C_{12} - (2^{1/2}/3)(\nu\nu'' - \nu')C_{13} \\
&- (6^{1/2}/27)(\nu\nu'' - \nu')C_{14} + (1/9)\gamma C_{16}\} \\
0 = &- w_r C_9 + (\kappa_r/\kappa)[5(6^{1/2}/6)(\nu C_3 - C_4) + (3/4)(\nu C_5 - C_7) \\
&- 7(3^{1/2}/12)(\nu C_6 - C_8) + (3^{1/2}/3)C_{15}] \\
&+ (\kappa_\rho/\kappa)[5(6^{1/2}/6)(C_3 - \nu C_4) - (1/2)(C_5 - \nu C_7) \\
&+ (3^{1/2}/6)(C_6 - \nu C_8) + (3^{1/2}/3)C_{16}] \\
&+ k[-\gamma C_1 - (3^{1/2}/9)\gamma C_2 - 5(6^{1/2}/18)(\nu'' C_3 - \nu' C_4) \\
&- (1/2)(\nu'' C_5 - \nu' C_7) - (3^{1/2}/18)(\nu'' C_6 - \nu' C_8)] \\
&+ k(1-\nu^2)^{-1}\{(6^{1/2}/9)\gamma(C_{10} + C_{11}) - 2(5^{1/2}/9)\gamma\nu C_{12} \\
&+ (3^{1/2}/9)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}
\end{aligned}
$$

$$0 = -w_r C_{10} + (\kappa_r/\kappa)[-(2/3)C_3 + (6^{1/2}/2)C_5 - 7(2^{1/2}/6)C_6]$$

$$+ (\kappa_\rho/\kappa)[-(1/3)(3\nu C_3 - C_4) - (6^{1/2}/6)(3\nu C_5 - C_7)$$

$$+ (2^{1/2}/6)(3\nu C_6 - C_8) - (2^{1/2})C_{15}]$$

$$+ k[(1/3)\nu' C_3 - (6^{1/2}/2)\nu' C_5 - (2^{1/2}/6)\nu' C_6]$$

$$+ k(1-\nu^2)^{-1}\{(1/9)[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4]$$

$$-(6^{1/2}/6)[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7]$$

$$-(2^{1/2}/18)[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8]$$

$$-(6^{1/2}/27)\gamma C_9 + (2/3)\gamma\nu C_{10} - (30^{1/2}/9)\gamma C_{12}$$

$$+(1/3)\gamma C_{13} + (3^{1/2}/27)\gamma C_{14} - (2^{1/2}/3)(\nu\nu' - \nu'')C_{15}\}$$

$$0 = -w_r C_{11} + (\kappa_r/\kappa)[(1/3)(C_3 - 3\nu C_4) - (6^{1/2}/4)(C_5 - 3\nu C_7)$$

$$+7(2^{1/2}/12)(C_6 - 3\nu C_8) + (2^{1/2})C_{16}]$$

$$+ (\kappa_\rho/\kappa)[-(2/3)C_4 - (6^{1/2}/3)C_7 + (2^{1/2}/3)C_8]$$

$$+ k[(1/3)\nu'' C_4 - (6^{1/2}/2)\nu'' C_7 - (2^{1/2}/6)\nu'' C_8]$$

$$+ k(1-\nu^2)^{-1}\{(1/9)[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4]$$

$$-(6^{1/2}/6)[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7]$$

$$-(2^{1/2}/18)[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8]$$

$$-(6^{1/2}/27)\gamma C_9 - (2/3)\gamma\nu C_{11} + (30^{1/2}/9)\gamma C_{12}$$

$$+(1/3)\gamma C_{13} + (3^{1/2}/27)\gamma C_{14} + (2^{1/2}/3)(\nu\nu'' - \nu')C_{16}\}$$

$$
\begin{aligned}
0 = &- w_r C_{12} + (\kappa_r/\kappa)\big[-(30^{1/2}/30)(\nu C_3 + 3C_4) \\
&+ 3(5^{1/2}/20)(\nu C_5 + 3C_7) - 7(15^{1/2}/60)(\nu C_6 + 3C_8) \\
&+ (15^{1/2}/5)C_{15}\big] \\
&+ (\kappa_\rho/\kappa)\big[-(30^{1/2}/30)(3C_3 + \nu C_4) - (5^{1/2}/10)(3C_5 + \nu C_7) \\
&+ (15^{1/2}/30)(3C_6 + \nu C_8) - (15^{1/2}/5)C_{16}\big] \\
&+ k\big[(30^{1/2}/30)(\nu'' C_3 + \nu' C_4) - 3(5^{1/2}/10)(\nu'' C_5 + \nu' C_7) \\
&- (15^{1/2}/30)(\nu'' C_6 + \nu' C_8)\big] \\
&+ k(1-\nu^2)^{-1}\{(30^{1/2}/45)\nu[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4] \\
&- (5^{1/2}/5)\nu[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7] \\
&- (15^{1/2}/45)\nu[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8] \\
&- 2(5^{1/2}/45)\gamma\nu C_9 + (30^{1/2}/15)\gamma(C_{10} - C_{11}) \\
&+ (30^{1/2}/15)\gamma\nu C_{13} + (10^{1/2}/45)\gamma\nu C_{14} \\
&+ (15^{1/2}/15)[(\nu\nu'' - \nu')C_{15} - (\nu\nu' - \nu'')C_{16}]\}
\end{aligned}
$$

$$
\begin{aligned}
0 = &- w_r C_{13} + (\kappa_r/\kappa)\big[(3/4)(\nu C_3 - C_4) - (6^{1/2}/4)(\nu C_5 - C_7) \\
&+ 3(2^{1/2}/4)(\nu C_6 - C_8) - 3(2^{1/2}/4)C_{15}\big] \\
&+ (\kappa_\rho/\kappa)\big[-(1/2)(C_3 - \nu C_4) + (6^{1/2}/2)(C_5 - \nu C_7) \\
&- (2^{1/2}/2)(C_6 - \nu C_8) + (2^{1/2}/2)C_{16}\big] \\
&+ k\big[(6^{1/2}/6)\gamma C_1 + (2^{1/2}/2)\gamma C_2 - (1/2)(\nu'' C_3 - \nu' C_4) \\
&- (6^{1/2}/6)(\nu'' C_5 - \nu' C_7) - (2^{1/2}/2)(\nu'' C_6 - \nu' C_8)\big] \\
&+ k(1-\nu^2)^{-1}\{-\gamma(C_{10} + C_{11}) + (30^{1/2}/3)\gamma\nu C_{12} \\
&- (2^{1/2}/2)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}
\end{aligned}
$$

$$0 = -w_r C_{14} + (\kappa_r/\kappa)[-7(3^{1/2}/12)(\nu C_3 - C_4) + 3(2^{1/2}/4)(\nu C_5 - C_7)$$

$$+ 11(6^{1/2}/12)(\nu C_6 - C_8) + 7(6^{1/2}/12)C_{15}]$$

$$+ (\kappa_\rho/\kappa)[(3^{1/2}/6)(C_3 - \nu C_4) - (2^{1/2}/2)(C_5 - \nu C_7)$$

$$+ (6^{1/2}/6)(C_6 - \nu C_8) - (6^{1/2}/6)C_{16}]$$

$$+ k[(2^{1/2}/2)\gamma C_1 + (6^{1/2}/18)\gamma C_2 - (3^{1/2}/18)(\nu'' C_3 - \nu' C_4)$$

$$- (2^{1/2}/2)(\nu'' C_5 - \nu' C_7) - (6^{1/2}/18)(\nu'' C_6 - \nu' C_8)]$$

$$+ k(1-\nu^2)^{-1}\{-(3^{1/2}/9)\gamma(C_{10} + C_{11}) + (10^{1/2}/9)\gamma\nu C_{12}$$

$$- (6^{1/2}/18)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}$$

$$0 = -w_r C_{15} + (\kappa_r/\kappa)[(3^{1/2}/3)C_9 - (2^{1/2})\nu C_{10}$$

$$+ (15^{1/2}/3)C_{12} - 3(2^{1/2}/4)C_{13} + 7(6^{1/2}/12)C_{14}]$$

$$+ (\kappa_\rho/\kappa)[(3^{1/2}/3)\nu C_9 - (2^{1/2})(2C_{10} + C_{11})$$

$$+ (15^{1/2})\nu C_{12} + (2^{1/2}/2)\nu C_{13} - (6^{1/2}/6)\nu C_{14}]$$

$$+ k[(2^{1/2}/6)\gamma C_3 - (3^{1/2}/2)\gamma C_5 - (1/6)\gamma C_6 - (3^{1/2}/9)\nu' C_9$$

$$+ 2(2^{1/2}/3)\nu'' C_{10} - 2(15^{1/2}/9)\nu' C_{12} + (2^{1/2}/2)\nu' C_{13}$$

$$+ (6^{1/2}/18)\nu' C_{14}]$$

$$+ k(1-\nu^2)^{-1}\{(2^{1/2}/3)[\nu(\nu\nu'' - \nu')C_{10} - (\nu\nu' - \nu'')C_{11}]$$

$$+ (15^{1/2}/9)[\nu(\nu\nu' - \nu'') - (\nu\nu'' - \nu')]C_{12}$$

$$+ (1/3)\gamma(\nu C_{15} - C_{16})\}$$

$$0 = -w_r C_{16} + (\kappa_r/\kappa)\big[(3^{1/2}/3)\nu C_9 + (2^{1/2})(C_{10} + 2C_{11})$$
$$-(15^{1/2})\nu C_{12} - 3(2^{1/2}/4)\nu C_{13} + 7(6^{1/2}/12)\nu C_{14}\big]$$
$$+(\kappa_\rho/\kappa)\big[(3^{1/2}/3)C_9 + (2^{1/2})\nu C_{11} - (15^{1/2}/3)C_{12}$$
$$+(2^{1/2}/2)C_{13} - (6^{1/2}/6)C_{14}\big]$$
$$+k\big[(2^{1/2}/6)\gamma C_4 - (3^{1/2}/2)\gamma C_7 - (1/6)\gamma C_8$$
$$-(3^{1/2}/9)\nu'' C_9 - 2(2^{1/2}/3)\nu' C_{11} + 2(15^{1/2}/9)\nu'' C_{12}$$
$$+(2^{1/2}/2)\nu'' C_{13} + (6^{1/2}/18)\nu'' C_{14}\big]$$
$$+k(1-\nu^2)^{-1}\big\{(2^{1/2}/3)\big[(\nu\nu'' - \nu')C_{10} - \nu(\nu\nu' - \nu'')C_{11}\big]$$
$$+(15^{1/2}/9)\big[(\nu\nu' - \nu'') - \nu(\nu\nu'' - \nu')\big]C_{12}$$
$$+(1/3)\gamma(C_{15} - \nu C_{16})\big\}$$

$$0 = -w_\rho C_1 + (\kappa_r/\kappa)\big[(6^{1/2}/6)C_3 + (1/2)C_5 - (3^{1/2}/6)C_6\big]$$
$$+(\kappa_\rho/\kappa)\big[(6^{1/2}/3)C_4 - C_7 - (3^{1/2}/3)C_8\big]$$
$$+k(1-\nu^2)^{-1}\big\{-(6^{1/2}/9)\big[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\big]$$
$$-(1/3)\big[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7\big]$$
$$+(3^{1/2}/9)\big[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\big]$$
$$+(2/9)\gamma C_9 + (6^{1/2}/9)\gamma C_{13} - (2^{1/2}/9)\gamma C_{14}\big\}$$

$$0 = -w_\rho C_2 + (\kappa_r/\kappa)\big[-(2^{1/2}/6)C_3 - (3^{1/2}/6)C_5 + (1/6)C_6\big]$$
$$+(\kappa_\rho/\kappa)\big[(2^{1/2})C_4 - (3^{1/2}/3)C_7 + C_8\big]$$
$$+k(1-\nu^2)^{-1}\big\{(2^{1/2}/9)\big[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4\big]$$
$$+(3^{1/2}/9)\big[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7\big]$$
$$-(1/9)\big[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8\big]$$
$$-2(3^{1/2}/27)\gamma C_9 - (2^{1/2}/9)\gamma C_{13} + (6^{1/2}/27)\gamma C_{14}\big\}$$

$$0 = -w_\rho C_3 + (\kappa_r/\kappa)\big[(6^{1/2}/6)C_1 - (2^{1/2}/6)C_2 - (1/3)C_{10}\big]$$

$$+ (\kappa_\rho/\kappa)\big[(6^{1/2}/3)\nu C_1 + (2^{1/2})\nu C_2 - (1/3)C_{13} - (3^{1/2}/3)C_{14}\big]$$

$$+ k\big[(6^{1/2}/9)\nu' C_1 - (2^{1/2}/9)\nu' C_2\big]$$

$$+ k(1-\nu^2)^{-1}\{-(5/9)\gamma(\nu C_3 - C_4) + (6^{1/2}/18)\gamma(\nu C_5 - C_7)$$

$$-(2^{1/2}/18)\gamma(\nu C_6 - C_8) - 5(6^{1/2}/27)(\nu\nu' - \nu'')C_9$$

$$+(2/9)(\nu\nu'' - \nu')C_{10} + (30^{1/2}/27)(\nu\nu' - \nu'')C_{12}$$

$$+(1/9)(\nu\nu' - \nu'')C_{13} - (3^{1/2}/27)(\nu\nu' - \nu'')C_{14}$$

$$+(2^{1/2}/9)\gamma C_{15}\}$$

$$0 = -w_\rho C_4 + (\kappa_r/\kappa)\big[(6^{1/2}/6)\nu C_1 - (2^{1/2}/6)\nu C_2$$

$$-5(6^{1/2}/18)C_9 - (30^{1/2}/18)C_{12} + (1/6)C_{13}$$

$$-(3^{1/2}/18)C_{14}\big]$$

$$+ (\kappa_\rho/\kappa)\big[(6^{1/2}/3)C_1 + (2^{1/2})C_2\big]$$

$$+ k\big[(6^{1/2}/9)\nu'' C_1 - (2^{1/2}/9)\nu'' C_2\big]$$

$$+ k(1-\nu^2)^{-1}\{-(5/9)\gamma(C_3 - \nu C_4) + (6^{1/2}/18)\gamma(C_5 - \nu C_7)$$

$$-(2^{1/2}/18)\gamma(C_6 - \nu C_8) + 5(6^{1/2}/27)(\nu\nu'' - \nu')C_9$$

$$+(2/9)(\nu\nu' - \nu'')C_{11} + (30^{1/2}/27)(\nu\nu'' - \nu')C_{12}$$

$$-(1/9)(\nu\nu'' - \nu')C_{13} + (3^{1/2}/27)(\nu\nu'' - \nu')C_{14}$$

$$+(2^{1/2}/9)\gamma C_{16}\}$$

$$0 = -w_\rho C_5 + (\kappa_r/\kappa)\left[(1/2)C_1 - (3^{1/2}/6)C_2 - (6^{1/2}/6)C_{10}\right]$$

$$+ (\kappa_\rho/\kappa)\left[-\nu C_1 - (3^{1/2}/3)\nu C_2 - (1/3)C_9\right.$$

$$\left. - (5^{1/2}/3)C_{12} - (6^{1/2}/3)C_{13} - (2^{1/2}/3)C_{14}\right]$$

$$+ k\left[(1/3)\nu' C_1 - (3^{1/2}/9)\nu' C_2\right]$$

$$+ k(1-\nu^2)^{-1}\left\{(6^{1/2}/18)\gamma(\nu C_3 - C_4)\right.$$

$$- (1/3)\gamma(\nu C_5 - C_7) + (3^{1/2}/9)\gamma(\nu C_6 - C_8)$$

$$+ (1/9)(\nu\nu' - \nu'')C_9 + (6^{1/2}/9)(\nu\nu'' - \nu')C_{10}$$

$$+ (5^{1/2}/9)(\nu\nu' - \nu'')C_{12} - (6^{1/2}/9)(\nu\nu' - \nu'')C_{13}$$

$$\left. + (2^{1/2}/9)(\nu\nu' - \nu'')C_{14} + (3^{1/2}/9)\gamma C_{15}\right\}$$

$$0 = -w_\rho C_6 + (\kappa_r/\kappa)\left[-(3^{1/2}/6)C_1 + (1/6)C_2 + (2^{1/2}/6)C_{10}\right]$$

$$+ (\kappa_\rho/\kappa)\left[-(3^{1/2}/3)\nu C_1 + \nu C_2 - (3^{1/2}/3)C_9\right.$$

$$\left. - (15^{1/2}/3)C_{12} - (2^{1/2}/3)C_{13} + (6^{1/2}/3)C_{14}\right]$$

$$+ k\left[-(3^{1/2}/9)\nu' C_1 + (1/9)\nu' C_2\right]$$

$$+ k(1-\nu^2)^{-1}\left\{-(2^{1/2}/18)\gamma(\nu C_3 - C_4)\right.$$

$$+ (3^{1/2}/9)\gamma(\nu C_5 - C_7) - (1/9)\gamma(\nu C_6 - C_8)$$

$$- (3^{1/2}/27)(\nu\nu' - \nu'')C_9 - (2^{1/2}/9)(\nu\nu'' - \nu')C_{10}$$

$$- (15^{1/2}/27)(\nu\nu' - \nu'')C_{12} + (2^{1/2}/9)(\nu\nu' - \nu'')C_{13}$$

$$\left. - (6^{1/2}/27)(\nu\nu' - \nu'')C_{14} - (1/9)\gamma C_{15}\right\}$$

$$0 = -w_\rho C_7 + (\kappa_r/\kappa)[(1/2)\nu C_1 - (3^{1/2}/6)\nu C_2 + (1/6)C_9$$
$$-(5^{1/2}/6)C_{12} - (6^{1/2}/6)C_{13} + (2^{1/2}/6)C_{14}]$$
$$+(\kappa_\rho/\kappa)[-C_1 - (3^{1/2}/3)C_2 - (6^{1/2}/3)C_{11}]$$
$$+k[(1/3)\nu''C_1 - (3^{1/2}/9)\nu''C_2]$$
$$+k(1-\nu^2)^{-1}\{(6^{1/2}/18)\gamma(C_3 - \nu C_4)$$
$$-(1/3)\gamma(C_5 - \nu C_7) + (3^{1/2}/9)\gamma(C_6 - \nu C_8)$$
$$-(1/9)(\nu\nu'' - \nu')C_9 + (6^{1/2}/9)(\nu\nu' - \nu'')C_{11}$$
$$+(5^{1/2}/9)(\nu\nu'' - \nu')C_{12} + (6^{1/2}/9)(\nu\nu'' - \nu')C_{13}$$
$$-(2^{1/2}/9)(\nu\nu'' - \nu')C_{14} + (3^{1/2}/9)\gamma C_{16}\}$$

$$0 = -w_\rho C_8 + (\kappa_r/\kappa)[-(3^{1/2}/6)\nu C_1 + (1/6)\nu C_2$$
$$-(3^{1/2}/18)C_9 + (15^{1/2}/18)C_{12} + (2^{1/2}/6)C_{13}$$
$$-(6^{1/2}/18)C_{14}]$$
$$+(\kappa_\rho/\kappa)[-(3^{1/2}/3)C_1 + C_2 - (2^{1/2})C_{11}]$$
$$+k[-(3^{1/2}/9)\nu''C_1 + (1/9)\nu''C_2]$$
$$+k(1-\nu^2)^{-1}\{-(2^{1/2}/18)\gamma(C_3 - \nu C_4)$$
$$+(3^{1/2}/9)\gamma(C_5 - \nu C_7) - (1/9)\gamma(C_6 - \nu C_8)$$
$$+(3^{1/2}/27)(\nu\nu'' - \nu')C_9 - (2^{1/2}/9)(\nu\nu' - \nu'')C_{11}$$
$$-(15^{1/2}/27)(\nu\nu'' - \nu')C_{12} - (2^{1/2}/9)(\nu\nu'' - \nu')C_{13}$$
$$+(6^{1/2}/27)(\nu\nu'' - \nu')C_{14} - (1/9)\gamma C_{16}\}$$

$$0 = -w_\rho C_9 + (\kappa_r/\kappa)[5(6^{1/2}/12)(\nu C_3 - C_4) - (1/4)(\nu C_5 - C_7)$$
$$+ (3^{1/2}/12)(\nu C_6 - C_8) + (3^{1/2}/6)C_{15}]$$
$$+ (\kappa_\rho/\kappa)[-(1/2)(C_5 - \nu C_7) - (3^{1/2}/2)(C_6 - \nu C_8)]$$
$$+ k[-(1/3)\gamma C_1 + (3^{1/2}/9)\gamma C_2 + 5(6^{1/2}/18)(\nu'' C_3 - \nu' C_4)$$
$$-(1/6)(\nu'' C_5 - \nu' C_7) + (3^{1/2}/18)(\nu'' C_6 - \nu' C_8)]$$
$$+ k(1-\nu^2)^{-1}\{-(6^{1/2}/9)\gamma(C_{10} + C_{11}) + 2(5^{1/2}/9)\gamma\nu C_{12}$$
$$-(3^{1/2}/9)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}$$

$$0 = -w_\rho C_{10} + (\kappa_r/\kappa)[-(1/3)C_3 - (6^{1/2}/6)C_5 + (2^{1/2}/6)C_6]$$
$$+ (\kappa_\rho/\kappa)[-(6^{1/2}/6)(3\nu C_5 - C_7) - (2^{1/2}/2)(3\nu C_6 - C_8)]$$
$$+ k[-(1/3)\nu' C_3 - (6^{1/2}/6)\nu' C_5 + (2^{1/2}/6)\nu' C_6]$$
$$+ k(1-\nu^2)^{-1}\{-(1/9)[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4]$$
$$-(6^{1/2}/18)[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7]$$
$$+ (2^{1/2}/18)[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8]$$
$$+ (6^{1/2}/27)\gamma C_9 - (2/3)\gamma\nu C_{10} + (30^{1/2}/9)\gamma C_{12}$$
$$+ (1/9)\gamma C_{13} - (3^{1/2}/27)\gamma C_{14} + (2^{1/2}/3)(\nu\nu' - \nu'')C_{15}\}$$

$$0 = -w_\rho C_{11} + (\kappa_r/\kappa)[(1/6)(C_3 - 3\nu C_4)$$
$$+ (6^{1/2}/12)(C_5 - 3\nu C_7) - (2^{1/2}/12)(C_6 - 3\nu C_8)$$
$$+ (2^{1/2}/2)C_{16}]$$
$$+ (\kappa_\rho/\kappa)[-(6^{1/2}/3)C_7 - (2^{1/2})C_8]$$
$$+ k[-(1/3)\nu'' C_4 - (6^{1/2}/6)\nu'' C_7 + (2^{1/2}/6)\nu'' C_8]$$
$$+ k(1-\nu^2)^{-1}\{-(1/9)[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4]$$
$$-(6^{1/2}/18)[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7]$$
$$+ (2^{1/2}/18)[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8]$$
$$+ (6^{1/2}/27)\gamma C_9 + (2/3)\gamma\nu C_{11} - (30^{1/2}/9)\gamma C_{12}$$
$$+ (1/9)\gamma C_{13} - (3^{1/2}/27)\gamma C_{14} - (2^{1/2}/3)(\nu\nu'' - \nu')C_{16}\}$$

$$
\begin{aligned}
0 = & - w_\rho C_{12} + (\kappa_r/\kappa)[-(30^{1/2}/60)(\nu C_3 + 3C_4) \\
& - (5^{1/2}/20)(\nu C_5 + 3C_7) + (15^{1/2}/60)(\nu C_6 + 3C_8) \\
& + (15^{1/2}/10) C_{15}] \\
& + (\kappa_\rho/\kappa)[-(5^{1/2}/10)(3C_5 + \nu C_7) - (15^{1/2}/10)(3C_6 + \nu C_8)] \\
& + k[-(30^{1/2}/30)(\nu'' C_3 + \nu' C_4) - (5^{1/2}/10)(\nu'' C_5 + \nu' C_7) \\
& + (15^{1/2}/30)(\nu'' C_6 + \nu' C_8)] \\
& + k(1-\nu^2)^{-1}\{-(30^{1/2}/45)\nu[(\nu\nu'' - \nu')C_3 + (\nu\nu' - \nu'')C_4] \\
& - (5^{1/2}/15)\nu[(\nu\nu'' - \nu')C_5 + (\nu\nu' - \nu'')C_7] \\
& + (15^{1/2}/45)\nu[(\nu\nu'' - \nu')C_6 + (\nu\nu' - \nu'')C_8] \\
& + 2(5^{1/2}/45)\gamma\nu C_9 - (30^{1/2}/15)\gamma(C_{10} - C_{11}) \\
& + (30^{1/2}/45)\gamma\nu C_{13} - (10^{1/2}/45)\gamma\nu C_{14} \\
& - (15^{1/2}/15)[(\nu\nu'' - \nu')C_{15} - (\nu\nu' - \nu'')C_{16}]\}
\end{aligned}
$$

$$
\begin{aligned}
0 = & - w_\rho C_{13} + (\kappa_r/\kappa)[-(1/4)(\nu C_3 - C_4) + (6^{1/2}/4)(\nu C_5 - C_7) \\
& - (2^{1/2}/4)(\nu C_6 - C_8) + (2^{1/2}/4) C_{15}] \\
& + (\kappa_\rho/\kappa)[-(1/2)(C_3 - \nu C_4) - (6^{1/2}/2)(C_5 - \nu C_7) \\
& - (2^{1/2}/2)(C_6 - \nu C_8) + (2^{1/2}/2) C_{16}] \\
& + k[-(6^{1/2}/6)\gamma C_1 + (2^{1/2}/6)\gamma C_2 - (1/6)(\nu'' C_3 - \nu' C_4) \\
& + (6^{1/2}/6)(\nu'' C_5 - \nu' C_7) - (2^{1/2}/6)(\nu'' C_6 - \nu' C_8)] \\
& + k(1-\nu^2)^{-1}\{-(1/3)\gamma(C_{10} + C_{11}) + (30^{1/2}/9)\gamma\nu C_{12} \\
& - (2^{1/2}/6)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}
\end{aligned}
$$

$$0 = - w_\rho C_{14} + (\kappa_r/\kappa)[(3^{1/2}/12)(\nu C_3 - C_4) - (2^{1/2}/4)(\nu C_5 - C_7)$$

$$+ (6^{1/2}/12)(\nu C_6 - C_8) - (6^{1/2}/12) C_{15}]$$

$$+ (\kappa_\rho/\kappa)[- (3^{1/2}/2)(C_3 - \nu C_4) - (2^{1/2}/2)(C_5 - \nu C_7)$$

$$+ (6^{1/2}/2)(C_6 - \nu C_8) + (6^{1/2}/2) C_{16}]$$

$$+ k[(2^{1/2}/6)\gamma C_1 - (6^{1/2}/18)\gamma C_2 + (3^{1/2}/18)(\nu'' C_3 - \nu' C_4)$$

$$- (2^{1/2}/6)(\nu'' C_5 - \nu' C_7) + (6^{1/2}/18)(\nu'' C_6 - \nu' C_8)]$$

$$+ k(1-\nu^2)^{-1}\{(3^{1/2}/9)\gamma(C_{10} + C_{11}) - (10^{1/2}/9)\gamma\nu C_{12}$$

$$+ (6^{1/2}/18)[(\nu\nu'' - \nu')C_{15} + (\nu\nu' - \nu'')C_{16}]\}$$

$$0 = - w_\rho C_{15} + (\kappa_r/\kappa)[(3^{1/2}/6) C_9 - (2^{1/2}/2)\nu C_{10}$$

$$+ (15^{1/2}/6) C_{12} + (2^{1/2}/4) C_{13} - (6^{1/2}/12) C_{14}]$$

$$+ (\kappa_\rho/\kappa)[(2^{1/2}/2)\nu C_{13} + (6^{1/2}/2)\nu C_{14}]$$

$$+ k[- (2^{1/2}/6)\gamma C_3 - (3^{1/2}/6)\gamma C_5 + (1/6)\gamma C_6$$

$$+ (3^{1/2}/9)\nu' C_9 - 2(2^{1/2}/3)\nu'' C_{10} + 2(15^{1/2}/9)\nu' C_{12}$$

$$+ (2^{1/2}/6)\nu' C_{13} - (6^{1/2}/18)\nu' C_{14}]$$

$$+ k(1-\nu^2)^{-1}\{ - (2^{1/2}/3)[\nu(\nu\nu'' - \nu')C_{10} - (\nu\nu' - \nu'')C_{11}]$$

$$- (15^{1/2}/9)[\nu(\nu\nu' - \nu'') - (\nu\nu'' - \nu')] C_{12}$$

$$- (1/3)\gamma(\nu C_{15} - C_{16})\}$$

$$\begin{aligned}
0 = &- w_\rho C_{16} + (\kappa_r/\kappa)\left[(3^{1/2}/6)\nu C_9 + (2^{1/2}/2)(C_{10} + 2C_{11})\right.\\
&\left. - (15^{1/2}/2)\nu C_{12} + (2^{1/2}/4)\nu C_{13} - (6^{1/2}/12)\nu C_{14}\right]\\
&+ (\kappa_\rho/\kappa)\left[(2^{1/2}/2)C_{13} + (6^{1/2}/2)C_{14}\right]\\
&+ k\left[-(2^{1/2}/6)\gamma C_4 - (3^{1/2}/6)\gamma C_7 + (1/6)\gamma C_8\right.\\
&+ (3^{1/2}/9)\nu'' C_9 + 2(2^{1/2}/3)\nu' C_{11} - 2(15^{1/2}/9)\nu'' C_{12}\\
&\left. + (2^{1/2}/6)\nu'' C_{13} - (6^{1/2}/18)\nu'' C_{14}\right]\\
&+ k(1-\nu^2)^{-1}\left\{-(2^{1/2}/3)\left[(\nu\nu'' - \nu')C_{10} - \nu(\nu\nu' - \nu'')C_{11}\right]\right.\\
&- (15^{1/2}/9)\left[(\nu\nu' - \nu'') - \nu(\nu\nu'' - \nu')\right]C_{12}\\
&\left. - (1/3)\gamma(C_{15} - \nu C_{16})\right\}
\end{aligned}$$

ACKNOWLEDGMENTS

Early portions of the trilocal analysis were carried out with the support of the U.S. Office of Naval Research, through Contract Nonr-778(00), 1952–54. During 1969 the trilocal analysis was supported by a Research Corporation grant.

REFERENCES

Clapp, R. E. (1980). Nonlocal structures: Bilocal photon, *International Journal of Theoretical Physics*, **19**, 31–88.

Clapp, R. E., Mack, E. W., Simons, F., and Wolf, J. A., Jr. (1980). Trilocal structures. I. Secular equation, *International Journal of Theoretical Physics*, **19**, 89–98.

Clapp, R. E., Mack, E. W., Simons, F., and Wolf, J. A., Jr. (1979). Trilocal structures. II. Expansion, *International Journal of Theoretical Physics*, **18**, 23–40.

Clapp, R. E., Mack, E. W., Simons, F., and Wolf, J. A., Jr. (1981a). Trilocal structures. III. Expansion in tree functions, *International Journal of Theoretical Physics*, **20**, 121–146.

Clapp, R. E., Mack, E. W., Simons, F., and Wolf, J. A., Jr. (1981b). Trilocal structures. IV. Momentum-dependent tree functions, *International Journal of Theoretical Physics*, **20**, 519–562.